USE OF THE DISCOVERY METHOD OF TEACHING LOW ACHIEVERS IN NINTH GRADE GENERAL MATHEMATICS

HOLLY HAMMOND FISHER

USE OF THE DISCOVERY METHOD OF TEACHING LOW ACHIEVERS IN NINTH GRADE GENERAL MATHEMATICS

An Abstract

Presented to

the Graduate Council of

Austin Peay State University

In Partial Fulfillment

of the Requirements for the Degree

Master of Arts

in Education

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by

Holly Hammond Fisher

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ABSTRACT

The purpose of this study was to examine under controlled conditions the effectiveness of the discovery method of teaching ninth grade general mathematics to low achievers. Comparison of two methods of teaching was accomplished through the preparation and the use of programmed materials based on the discovery method of teaching and programmed materials based on the traditional method of teaching. The study seemed justifiable in view of the need for better methods of educating low achievers. Materials used to teach these students must compensate for their deficiencies in reading and mathematics as well as to interest and motivate them. The related research indicated that the discovery method of teaching may be of some benefit to these low achieving students.

For this study, the following hypothesis was made: There is no significant difference in achievement in mathematics between low achievers taught with materials based on the discovery method of teaching and low achievers taught with materials based on the traditional method of teaching.

The experiment was conducted with two classes (sixty-two students) of ninth grade low achievers in mathematics at New Providence Junior High School, Clarksville, Tennessee. The author taught these classes from September, 1968 through March, 1969. These students were randomly assigned to two

groups. Early in March, 1969, the California Arithmetic Achievement Test was administered to the two classes to test for any significant difference in mathematical ability between the experimental group and the control group. On March 14, 1969 the pre test was administered to the subjects to test for any previous knowledge of exponents. No students were eliminated because of prior knowledge of exponents.

For the experiment the students were taught by programmed units of instruction written by the author. They concerned the following laws of exponents:

1. For each natural number a, and for all natural numbers x and y, $a^x \cdot a^y = a^{x+y}$.

2. For each natural number a, and for all natural numbers x and y, $(a^x)^y = a^{xy}$.

The experimental group was taught by a program based on the discovery method of teaching in which the students were led towards the discovery of the laws by the structure of the questions and the discussions. The control group was taught by a program based on the traditional method of teaching which contained the laws along with practice exercises. The students, who received the programs on March 17, 1969, needed fifty minutes to independently complete them. Of the sixty-two students that had been assigned to the groups, only forty-six completed the study, the others being eliminated because they were absent on days during which the experiment was being conducted.

The scores of the groups on the California Arithmetic Achievement Test were examined by means of a t-test. The results were interpreted to mean that there was not a significant difference in the mathematical ability of the two groups at the beginning of the study. The scores of the post test were also examined by means of a t-test. The results of this analysis were interpreted to mean that the achievement of the students being taught by the discovery method of teaching was not significantly different from the achievement of the subjects taught by the traditional method and the null hypothesis must be accepted.

This study indicated that the discovery method does not have a greater effect than the traditional method on mathematical achievement.

USE OF THE DISCOVERY METHOD OF TEACHING LOW ACHIEVERS IN NINTH GRADE GENERAL MATHEMATICS

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by

Holly Hammond Fisher

August 1969

To the Graduate Council:

I am submitting herewith a Thesis written by Holly Hammond Fisher entitled "Use of the Discovery Method of Teaching Low Achievers in Ninth Grade General Mathematics." I recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Arts in Education, with a major in Mathematics.

William G. Stokes
Major Professor

We have read this thesis and recommend its acceptance:

Fred A. Bunger
Minor Professor

Ernest T. Woodward
Third Committee Member

Accepted for the Council:

Wayne E. Stamper
Dean of the Graduate School

ACKNOWLEDGEMENTS

This author wishes to express her gratitude to the three members of her graduate committee, Dr. William G. Stokes, Dr. Fred Bunger, and Dr. Ernest Woodward, for all their help, understanding and patience in making this study a reality. An expression of sincere appreciation goes to Mr. Clint Daniel, Principal of New Providence Junior High School, for allowing the use of the facilities of the school for the experiment. Finally this author wishes to thank her husband, John, for all his encouragement and help, without which the study would not have been completed.

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CHAPTER I

THE NATURE OF THE PROBLEM

I. INTRODUCTION

Within the regular classroom most teachers encounter a variety of ability levels among the students. Consequently, the teacher faces a difficult task when trying to meet the individual needs of all the students. Of particular interest in this study are the low achievers in mathematics.

One method of reducing the range of abilities in the classroom is homogeneous grouping by ability, although this alone is often ineffective. Materials are seldom adequate to meet the needs of low achievers. According to Kenneth Easterday, "Two frequent teacher criticisms made of commercially prepared mathematics textbooks are that the reading level of the texts is too difficult and that the language and mathematics are too rigorous."[1] He suggests that materials for low achievers should be written with a reading level well below the assigned grade level.

William G. Mehl agrees that one difficulty with teaching lower ability students is that of providing ade-

[1] Kenneth Easterday, "A Technique for Low Achievers," The Mathematics Teacher, LVIII (October, 1965), 519.

quate materials. Junior high school students do not like the remedial approach to mathematics regardless of how deficient they are in mathematical ability. Many teachers cope with the problem by using textbooks that are at least one year behind the actual grade level of the student. This lowers the prestige of the student and is therefore embarrassing, as well as degrading and boring, to the student.[2]

In addition to possessing mathematical ability below their specified grade level, these students are often typified by frequent failures which tend to cause them discouragement and lack of motivation. Therefore, materials used to teach these low achievers must be written not only to compensate for their deficiencies in reading and mathematics, but also to interest and motivate the student. Teaching techniques also need revision. It is the opinion of some that preparation of materials designed for guided discovery learning may be one possible approach in teaching the low achiever in mathematics.

Robert Glaser defines learning by discovery "as teaching an association, concept or rule which involves

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[2] William G. Mehl, "Providing for the Basic Student in the Junior High School," The Mathematics Teacher, LIII (May, 1960), 359.

'discovery' of the association, concept or rule."[3] Kersh and Wittrock speak of discovery as a learner heading towards a specific learning goal without help from the teacher.[4] However, discovery should not be equated with a laissez-faire educational philosophy. With discovery, the student's behavior is subject to control just as it is with the old-fashioned drill method. It is the pattern of control that differs.[5] In the drill method the student is repeatedly given the concept or principle along with practice. However, the discovery approach controls the student by leading him towards the discovery through discussions and questions built into the presentation of the material.

Jerome Bruner says that discovery is a process of working, not what is learned. Its principal benefit is the encouragement of such a process of working. Since some children approach learning as listeners while others approach learning as speakers, discovery is important for main-

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[3] Robert Glaser, "Variables in Discovery Learning" (in Learning by Discovery: A Critical Appraisal, ed. Lee S. Shulman and Evan R. Keislar. Chicago: Rand McNally and Company, 1966), 14-15.

[4] Bert Y. Kersh and Merl C. Wittrock, "Learning by Discovery: An Interpretation of Recent Research," The Journal of Teacher Education, XII (December, 1962), 461.

[5] Howard H. Kendler, "Reflections on the Conference" (in Learning by Discovery: A Critical Appraisal, ed. Lee S. Shulan and Evan R. Keislar. Chicago: Rand McNally and Company, 1966), 172.

taining a balanced educational atmosphere for both type of student.[6]

Advocates of discovery learning encourage this teaching method because they feel it offers certain advantages. Bruner claims that discovery benefits the learner in four ways:

1. the increase in intellectual potency
2. the shift from extrinsic to instrinsic rewards
3. learning the heuristics of discovery
4. the aid to memory processing.[7]

Jerome Kagan says that discovery learning is beneficial because it arouses the student's interest, and, therefore, increases his attention. Because the student has to exert more effort in learning, he attaches more value to the task. The discovery approach to learning allows the child to develop principles without abundant guidance from an external agent. Therefore, the student is given more freedom from the submissiveness which teachers usually expect of students.[8]

Christofferson speaks of three levels of teaching

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[6] Jerome S. Bruner, "On Learning Mathematics," The Mathematics Teacher, LIII (December, 1960), 612-613.

[7] Jerome S. Bruner, "The Act of Discovery," Harvard Educational Review, (Winter, 1961), 23.

[8] Jerome Kagan, "Learning, Attention, and the Issue of Discovery" (in Learning by Discovery: A Critical Appraisal, ed. Lee S. Shulman and Evan R. Keislar. Chicago: Rand McNally and Company, 1966), 158-159.

mathematics:

1. teaching the rule
2. telling the student everything, explaining every step, for him to understand, to remember, and to follow
3. creative thinking.[9]

He says that provision of creative experiences which lead the student towards the discovery of principles or relationships by his own efforts rather than having them told to him is becoming predominant in mathematics as well as in other areas.

However, there are certain disadvantages to the discovery method of teaching that should also be considered. It is important to remember that everything need not be discovered. This was explained by Wittrock who wrote, "One must learn to acquire and comprehend much of his culture as well as to discover new knowledge and to solve problems."[10]

Hendrix points out that misuse of the discovery learning technique by teachers can be detrimental to the student. They often call for the generalizations before the students have noticed any similarities between the examples,

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[9]H. C. Christofferson, "Creative Teaching in Mathematics," The Mathematics Teacher, LI (November, 1958), 535.

[10]M. C. Wittrock, "The Learning by Discovery Hypothesis" (in Learning by Discovery: A Critical Appraisal, ed. Lee S. Shulman and Evan R. Keislar. Chicago: Rand McNally and Company, 1966), 36.

or they may ask for generalizations without realizing that the students do not possess the necessary vocabulary or rules for sentence structure.[11]

II. THE PROBLEM

Statement of the Problem. The purpose of this study was to examine under controlled conditions the effectiveness of the discovery method of teaching ninth grade general mathematics to low achievers. Comparison of two methods of teaching was accomplished through the preparation and the use of programmed materials based on the discovery method of teaching and programmed materials based on the traditional method of teaching.

Delimitation. This study was limited to forty-six ninth grade students enrolled in general mathematics at New Providence Junior High School in Clarksville, Tennessee. These students were homogeneously grouped on the basis of intellectual ability and previous success in mathematics. Of the ninth grade general mathematics classes at New Providence Junior High School, these two groups were ranked as having the lowest mathematical ability.

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[11] Gertrude Hendrix, "Learning by Discovery," The Mathematics Teacher, LIV (May, 1961), 292.

Significance of the Study. This study is significant in that it is a continuation of research conducted to test the merit of discovery learning. The discovery method is one of a variety of teaching methods. In science, laboratory work may lead to discovery. In social studies or English, discovery of generalizations may be attained through guidance in class discussions. Should the guided discovery approach prove effective in teaching the low achiever a particular concept of algebra, it could provide teachers with some indication of one possible technique for teaching mathematics to students of this ability level. Also, this significance might possibly be extended to students at other ability levels and in other subject areas.

Hypothesis. For this study, the following hypothesis was made:

There is no significant difference in achievement in mathematics between low achievers taught with materials based on the discovery method of teaching and low achievers taught with materials based on the traditional method of teaching.

III. DEFINITION OF TERMS.

Much of the terminology of this study followed the accepted meanings found in standard reference works within the fields of mathematics and education. However, certain terminology used in the context of this study required specific definitions which are given below:

<u>Guided Discovery Method of Teaching</u>. The guided discovery method of teaching was defined as the method of teaching in which the student is directed toward an awareness of mathematical concepts by the structure of the materials.

<u>Traditional Method of Teaching</u>. The traditional method of teaching was defined as the method of teaching in which the student is provided with the mathematical concepts within the structure of the materials.

<u>Low Achievers in Mathematics</u>. Low achievers in mathematics were defined as students who are performing below their specified grade level in mathematics.

<u>Experimental Group</u>. The experimental group was defined as the group taught by the guided discovery method of teaching.

<u>Control Group</u>. The control group was taught by the traditional method of teaching.

<u>Pre Test</u>. The pre test was a brief test of five problems concerned with exponents administered on March 14, 1969 to both the experimental group and the control group. A copy of the pre test is located in Appendix C.

Post Test. The post test was a multiple choice test concerned with the two laws of exponents. It was written by the author and was administered on March 18, 1969 to both the experimental group and the control group. A copy of the post test is located in Appendix C.

IV. ORGANIZATION OF THE PAPER

The first chapter consists of the introduction, the statement of the problem, the delimitation, the significance of the study, the hypothesis, and the definitions of the terms. Chapter II contains a review of certain literature related to discovery learning in education. Chapter III is devoted to the methods of investigation that were used in conducting the study. Chapter IV is devoted to the statistical analysis of the data and the results. Chapter V is concerned with the conclusions of the study and gives recommendations for further study.

CHAPTER II

REVIEW OF RELATED LITERATURE

Much research has been done concerning discovery learning and the teaching of mathematics. In 1949 R. E. Michaels used ninth graders to test two methods of teaching the fundamental operations of positive and negative numbers and the solution of simple equations. Method A emphasized exercises in thinking, and through exercises built around familiar situations such as time, directions, money, etc., the student was expected to discover and understand the principles. In method B statements of rules of the operations were combined with extensive practice or drill, with no attempt to explain why the principles worked. The overall results of the study favored method B.[12]

Robert Craig used multiple choice verbal items with college students to test the effect of varying amounts of guidance in discovery learning. One group was the independent discovery group while the second group was the directed discovery group. The directed discovery group learned more relationships in each of the three trials but in general

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[12] R. E. Michaels, "The Relative Effectiveness of Two Methods of Teaching Certain Topics in Ninth Grade Algebra," *The Mathematics Teacher*, XLII (February, 1949), 83-87.

there was no difference in retention.[13]

According to Kittell, there are two extremes in educational practice: (1) the teaching-learning situation, and (2) the complete presentation of selected, specific principles and facts, the rote learning method. In a study made in 1957, he added a third alternative by adding some direction aids to the process of discovery learning. Ninth graders were divided into three groups, labeled Minimum, Intermediate, and Maximum, the difference being the amount of direction they received. The subjects were given verbal problems. The results seemed to indicate that the subjects from the Intermediate group learned and transferred as many or more principles than the subjects from the other two groups who received more or less direction.[14]

A study done by Max A. Sobel was intended to determine if there is any relationship between the learning of mathematical concepts and the method of presentation. Method A was an abstract verbalized deductive method where the concepts were defined and presented by the teacher.

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[13]Robert C. Craig, "Direct Versus Independent Discovery of Established Relations," Journal of Educational Psychology, XLVII (April, 1956), 223-234.

[14]Kack E. Kittell, "An Experimental Study of the Effect of External Direction During Learning in Transfer and Retention of Principles," Journal of Educational Psychology, XLVIII (November, 1957), 391-405.

Method B was a concrete, nonverbalized inductive method where the students were guided through experiences involving applications with the hope that they discover and verbalize concepts. Because of the difference in intellectual ability, the experimental and control groups were divided into two subgroups, those with high intelligence test scores and those with average intelligence test scores. At the 5% level of confidence the study favored the experimental subgroup with the high intellectual ability. There was no significant difference between the average groups. Sobel speculates that it would be interesting to perform a similar experiment with a third group having low intelligence test scores. He says that it is possible that those with low intellectual ability might favor the control method. This would contradict the advice of educators on handling the situation.[15]

In his experiment with the discovery method of teaching low achievers in tenth grade general mathematics, Jack Price used three groups. The control group was taught with a traditional textbook which was primarily deductive in nature. A second group was taught with materials designed to promote student discovery but followed the general out-

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[15]Max A. Sobel, "Concept Learning in Algebra," The Mathematics Teacher, XLIX (October, 1956), 425-439.

line of the textbook. Group three was taught with the same materials as group two, but with additional materials designed to aid the transfer of mathematical thinking. The two groups taught by the discovery techniques showed a slight gain over the control group in mathematical achievement. Price concluded that discovery techniques were beneficial for teaching the general mathematics students.[16]

At least two curriculum projects have developed materials based on the discovery method of teaching. The Madison Project is an extensive experimental teaching project under the leadership of Robert Davis at Syracuse University and Webster College, Webster Groves, Missouri. In describing the work of this group, Davis says, "The Madison Project materials are founded on the belief that good mathematics is somewhat akin to good jazz-it must be experienced, and the spirit is more important than the outward form."[17] Lectures, as well as written exposition, have been discarded and have been replaced with teachers serving as moderators or discussion leaders. One of the first uses of the materials was with junior high school students who were low achievers in

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[16] Jack Price, "Discovery: Its Effect in Critical Thinking and Achievement in Mathematics," The Mathematics Teacher, LX (December, 1967), 874-876.

[17] Robert B. Davis, Discovery in Mathematics (Reading, Massachusetts: Addison Wesley Publishing Company, Inc., 1964), 8.

mathematics. It was considered beneficial for these students because it required less use of language.[18]

William Johntz heads a project called Special Education for the Disadvantaged in which abstract algebra is taught to culturally deprived elementary school children. According to Johntz, the cause of low achievement is lack of motivation. An effective way to overcome this is to recreate a successful learning situation in a high-status subject. Mr. Johntz attempts to accomplish this with the discovery method of teaching.[19]

The preceding evidence is inconclusive regarding the effectiveness of the discovery method of teaching. Therefore, further investigation seems warranted to test its value when used with low achievers in mathematics.

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[18] Robert Davis, "The 'Madison Project' of Syracuse University," The Mathematics Teacher, LIII (November, 1960), 574.

[19] William F. Johntz, "Mathematics and the Culturally Disadvantaged" (in The Disadvantaged Learner, ed. Staten W. Webster. San Francisco: Chandler Publishing Company, 1966), 573-581; James Benet, "Googool," American Education, III (October, 1967), 9-10.

CHAPTER III

THE EXPERIMENT

The author taught two classes of ninth grade low achievers in mathematics at New Providence Junior High School from September, 1968 through March, 1969. One class was composed of thirty students, twenty-six boys and four girls. Thirteen boys and nineteen girls made up the other class. These students had been assigned to the classes on the basis of previous performance in mathematics, intelligence test scores, and achievement test scores. They were considered to have the lowest mathematical ability in the ninth grade.

Early in March, 1969, the California Arithmetic Achievement Test (Form W) was administered to the two classes. The scores were used to test for any significant difference in mathematical ability between the control group and the experimental group.

On March 14, 1969 the pre test was administered for the purpose of testing for any previous knowledge of exponents and, in particular, the two laws of exponents presented in the study. No one was eliminated from the study because of prior knowledge of exponents. The pre test and the other materials pertaining to the study were given to all students in the two classes. It was intended that the entire study be considered a regular classroom activity. The students were randomly assigned to two groups.

They were taught certain concepts of exponents with programmed units of instruction. In particular, the material concerned the following two laws of exponents:

1. For each natural number a and for all natural numbers x and y, $a^x \cdot a^y = a^{x+y}$.

2. For each natural number a and for all natural numbers x and y, $(a^x)^y = a^{xy}$.

Two programs were written by the experimenter. One was based on the traditional method of teaching where the laws were presented within the text of the program, followed by practice exercises. The second program was based on the discovery method of teaching. In the second program the actual laws were not given, but the students were led towards their discovery by the structure of the questions and the discussions. Verbalization was not required but the students were required to work problems based on the laws. A copy of each program may be found in Appendix B.

In the process of writing these programs, the author asked students from classes taught by other teachers to read them to insure their clarity. Any frames that were not clear were revised.

Prior to March 17, 1969, a group of ninth grade general mathematics students at New Providence Junior High

School being taught by another teacher had been provided with the programmed units. Multiple choice tests constructed by the author to cover the material in the programs were given to this group after they had completed the programs. Since the programmed units were short, content validity was established because the questions on the test were constructed so as to measure the ideas that the programs were intended to teach. The reliability coefficient of the post test, computed by split halves and the Spearman Brown Prophecy Formula, was 0.86.

The programs were given to the experimental and control groups on March 17, 1969. They had been arranged in booklets with one frame on a page. Instructions were given to the students by the experimenter explaining how the booklets were to be used, after which the students worked on the booklets independently. Approximately fifty minutes were required for completion of the programs. On March 18, 1969, the students had finished the programmed units and were given the post test. Of the sixty-two students that had been assigned to the two groups, only forty-six completed the study, the others being eliminated because they were absent on days during which the experiment was being conducted.

The post test scores were analyzed in order to determine the effectiveness of the discovery method of teaching as compared to the traditional method of teaching. A table of the test scores may be found in Appendix A.

CHAPTER IV

THE RESULTS

The subjects were given the California Arithmetic Achievement Test (Form W) to test the comparability of the two groups in mathematical ability. The difference between the means of the scores of the groups on the California Arithmetic Achievement Test was examined for significance by means of a t-test. The results are given in Table I. The value for t was 1.025. Since the value of t at the 5% level of confidence is greater than 2.041 for forty-four degrees of freedom, there was no significant difference between the means of the two groups. This was interpreted to mean there was not a significant difference in the mathematical ability of the two groups at the beginning of the study.

TABLE I

T-TEST BETWEEN THE CONTROL AND EXPERIMENTAL GROUPS FOR THE CALIFORNIA ARITHMETIC ACHIEVEMENT TEST

Group	Mean	σ	t
Experimental	67.174	9.707	1.025
Control	64.000	10.811	

The post test was administered on March 18, 1969 after the two groups had completed the programmed units of instruction. The difference between the means of the scores of the two groups on the post test was examined for significance by means of a t-test. The results are given in Table II. The value for t was 1.209. Since the value of t at the 5% level of confidence is greater than 2.041 for forty-four degrees of freedom, the achievement of the subjects being taught by the discovery method of teaching was not significantly different from the achievement of the subjects taught by the traditional method and the null hypothesis must be accepted.

TABLE II

T-TEST BETWEEN THE CONTROL AND EXPERIMENTAL GROUPS FOR THE POST TEST

Groups	Mean	σ	t
Experimental	14.870	5.543	1.209
Control	12.826	5.675	

CHAPTER V

SUMMARY AND CONCLUSIONS

The purpose of this study was to investigate the use of the discovery method of teaching with low achievers in ninth grade general mathematics. The study seemed justifiable in view of the need for better methods of educating low achieving students. The related research indicated that the discovery method of teaching may be of some benefit to these low achievers.

In March the students were randomly assigned to two groups. The students were then given the California Arithmetic Achievement Test to establish that the mathematical ability of the two groups at the beginning of the study was not significantly different. On March 17, 1969 the two groups were given the programmed units of instruction. After the students had completed these programs, the post test was administered. It was designed to measure the students' achievement of the concepts of exponents presented in the programmed units.

On the basis of the test scores, the following conclusion was reached:

Since the achievement of the subjects being taught by the discovery method of teaching was not significantly better (at the 5% level of confidence) than that of the subjects being taught by the traditional method of teaching, the discovery method does not have a greater effect than the traditional method on mathematical achievement.

The study did slightly favor the discovery method of teaching. However, this may be related to the slight initial differences in ability as measured by scores on the achievement test. Perhaps more conclusive results could have been established if the experimenter had included the four laws of exponents and the experiment had been extended over a longer period of time.

The questions of what materials should be provided for the low achievers in mathematics and how the guided discovery method of teaching may be used most effectively with these materials are questions that will depend upon further research for their answers. On the basis of this study, the following topics are suggested for further study:

1. The effectiveness of the use of the guided discovery method of teaching with low achievers in mathematics extended over a longer period of time.

2. The effectiveness of the use of the guided discovery method of teaching with students who have average or above average intelligence but who are low achievers in mathematics.

3. The effectiveness of the use of the guided discovery method of teaching in other areas of mathematics.

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APPENDIX A

APPENDIX A

TABLE III

TEST DATA

California Arithmetic Achievement Test		Post Test	
Experimental	Control	Experimental	Control
93	96	25	22
88	88	24	21
84	75	23	20
83	70	20	20
77	69	20	19
76	69	19	18
74	66	18	17
74	65	18	17
73	65	18	15
72	65	17	15
71	64	16	14
70	63	16	13
68	63	14	13
65	62	12	11
63	60	11	10
61	57	11	9
59	56	11	8
59	56	10	7
53	56	10	6
52	53	9	6
46	53	8	5
45	52	8	5
39	49	4	4
Mean 67.174	64.000	14.870	12.826

APPENDIX B

APPENDIX B

THE PROGRAMMED UNITS OF INSTRUCTION[20]

DIRECTIONS

This booklet represents both a textbook and a teacher. On each page material is presented, followed by questions. Read the material carefully and then answer the questions. When you have answered the questions, turn to the next page and check your answers. Continue in this way throughout the booklet. However, on some of the pages you may be directed to a page other than the one immediately following. It is, therefore, important for you to read all the directions carefully. Remember to read the material and answer the questions before checking the answers.

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[20] In writing the programs, the author referred to the following source: David L. Neuhauser, "A Comparison of Three Methods of Teaching a Programed Unit on Exponents to Eighth Grade Students" (unpublished Doctoral Dissertation, Florida State University, Tallahassee, 1964).

I. THE TRADITIONAL PROGRAM

1

In the expression $3 \cdot 3$, the 3 appears as a factor 2 times. A shorter way of writing this expression is 3^2. In the expression $2 \cdot 2 \cdot 2$, the 2 appears as a factor ________ times. A shorter way of writing the expression is 2^3.

2

Answer: 3

In the expression $3 \cdot 3 \cdot 3 \cdot 3$, the 3 appears as a factor 4 times. A shorter way of writing $3 \cdot 3 \cdot 3 \cdot 3$ is ________.

3

Answer: 3^4

We can write $2 \cdot 2$ as 2^2, and $2 \cdot 2 \cdot 2$ as 2^3. Then $2 \cdot 2 \cdot 2 \cdot 2$ = ________.

4

Answer: 2^4

$2^2 = 2 \cdot 2$

$2^3 = 2 \cdot 2 \cdot 2$

$2^4 = 2 \cdot 2 \cdot 2 \cdot 2$

$2^5 =$ ________

5

Answer: $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

$3 \cdot 3 = 3^2$

$2 \cdot 2 \cdot 2 = 2^3$

$4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 4^5$

Then $5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 =$ ____________.

6

Answer: 5^7

$2^4 = 2 \cdot 2 \cdot 2 \cdot 2$

$4^3 = 4 \cdot 4 \cdot 4$

$6^2 = 6 \cdot 6$

$3^5 =$ ______________

7

Answer: $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

Consider the expression $x \cdot x \cdot x$. In the expression, the x appears as a factor ___________ times. Another way of writing $x \cdot x \cdot x$ is ____________.

8

Answer: 3, x^3

Consider the expression x^5. In this expression, x appears as a factor ___________ times. Another way of writing x^5 is ______________.

9

Answer: 5, $x \cdot x \cdot x \cdot x \cdot x$

In the expression $2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$, the 2 is called the base. Thus in the expression $3^2 = 3 \cdot 3$, the 3 is called the ________.

10

Answer: Base

In the expression 3^4, the 4 is called the exponent. The base in the expression is ________. In the expression 4^2, the 2 is called the ________.

11

Answer: 3, exponent

Consider the expression $2^4 = 2 \cdot 2 \cdot 2 \cdot 2$. The base 2 is used as a factor 4 times. Four is also the exponent. Consider the expression $2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$. The base 2 is used as a factor ________ times. ________ is the exponent.

12

Answer: 5, 5

In the example $3^6 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$, the 3 is called the ________ and the 6 is called the ________.

13

Answer: Base, exponent

$4^3 = 4 \cdot 4 \cdot 4$

The base 4 is used as a factor 3 times. The exponent is 3. Thus the base is used as a ________ the same number of times as shown by the exponent.

14

Answer: factor

Then in the expression $x^3 = x \cdot x \cdot x$, the base x is used as a factor ________ times.

15

Answer: 3

Then in the expression $x^3 = x \cdot x \cdot x$, the base is used as a ________ 3 times. The ________ of the expression is 3.

16

Answer: factor, exponent

$x^4 = x \cdot x \cdot x \cdot x$

x is used as a factor ________ times and therefore the exponent is ________. $x^5 = x \cdot x \cdot x \cdot x \cdot x$

x is used as a ________ 5 times. Five is also the ________.

17

Answer: 4, 4, factor, exponent

Then in the expression $x^n = \underbrace{x \cdot x \cdot x \cdot \;.\;.\;.\;.\; x}_{n \text{ factors}}$, the base

x is used as a factor __________ times.

18

Answer: n

Then in the expression $x^m = x \cdot x \cdot x \cdot \;.\;.\;.\;.\; \cdot x$, the base x

is used as a ___________ m times. m is also the number

shown by the ____________ of the expression.

19

Answer: factor, exponent

The first powers of numbers is defined as follows:

1^1 is defined as 1.

2^1 is defined as 2.

3^1 is defined as ______.

20

Answer: 3

$4^2 = 4 \cdot 4$

$3^5 =$ ______ $\cdot$ ______ $\cdot$ ______ $\cdot$ ______ $\cdot$ ______

$1^3 =$ ______ $\cdot$ ______ $\cdot$ ______

21

Answer: $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$, $1 \cdot 1 \cdot 1$

$5^3 = 5 \cdot 5 \cdot 5$

$2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

$6^2 =$ __________

$x^5 =$ __________

22

Answer: $6 \cdot 6$, $x \cdot x \cdot x \cdot x \cdot x$

Written in simpler form: $2 \cdot 2 \cdot 2 = 2^3$

$3 \cdot 3 \cdot 3 \cdot 3 =$ __________

$4 \cdot 4 =$ __________

$x \cdot x \cdot x \cdot x \cdot x \cdot x =$ __________

23

Answer: 3^4, 4^2, x^6

A shorter way of writing $a \cdot a \cdot a \; . \; . \; . \; . \; . \; . \cdot a$ (n times) is

__________.

24

Answer: a^n

Rule 1: For each natural number a and for all natural numbers x and y, $a^x \cdot a^y = a^{x+y}$.

(Natural numbers is the set of numbers N = [1,2,3, . . .])

When we have a problem in the form $a^x \cdot a^y$, we find the product by adding the exponents.

$a^x \cdot a^y = a^{x+y}$

$a^2 \cdot a^3 = a^{2+3} = a^5$

$2^2 \cdot 2^3 = 2^{2+3} = 2^5$

$4^5 \cdot 4^6 = 4^{5+6} = 4^{11}$

$5^3 \cdot 5^4 =$ ________ = ________

25

Answer: 5^{3+4}, 5^7

Here are some more examples:

$2^5 \cdot 2^7 = 2^{5+7} = 2^{12}$

$3^4 \cdot 3^5 =$ ________

26

Answer: 3^9 (or 3^{4+5})

$6^2 \cdot 6^3 = 6^5$

$x^6 \cdot x^4 =$ ________

27

Answer: x^{10}

Now let us examine why rule 1 works.

In $5^3 = 5 \cdot 5 \cdot 5$, the 5 is used as a factor ________ times.

In $5^4 = 5 \cdot 5 \cdot 5 \cdot 5$, the 5 is used as a factor ________ times.

28

Answer: 3, 4

Then:

$$5^3 \cdot 5^4 = (\underbrace{5 \cdot 5 \cdot 5}_{3}) \cdot (\underbrace{5 \cdot 5 \cdot 5 \cdot 5}_{4})$$

$$3 + 4 = \underline{\qquad\qquad}$$

In the product, 5 is used as a factor ________ times.

29

Answer: 7, 7

In the expression $5^3 \cdot 5^4 = 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5$, the 5 is used as a factor 7 times.

Written in simpler form, $5^3 \cdot 5^4 = 5^{\underline{\quad} + \underline{\quad}} = 5^{\underline{\quad}}$.

30

Answer: 3, 4, 7

Consider the product $4^2 \cdot 4^3$.

$4^2 = 4 \cdot 4$

$4^3 = 4 \cdot 4 \cdot 4$

$4^2 \cdot 4^3 = (4 \cdot 4) \cdot (4 \cdot 4 \cdot 4) = 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 4^{\underline{\quad}}$

31

Answer: 5

In the expression $4^2 = 4 \cdot 4$, the 4 appears as a factor

__________ times.

In the expression $4^3 = 4 \cdot 4 \cdot 4$, the 4 appears as a factor

__________ times.

Therefore, in the product $4^2 \cdot 4^3$, the 4 appears as a

factor ________ + ________ = 5 times.

32

Answer: 2, 3, 2, 3

Then from rule 1 we know that $4^2 \cdot 4^3 = 4^{\_\_\_ + \_\_\_} = 4^{\_\_\_\_}$.

33

Answer: 2, 3, 5

1. $10^5 \cdot 10^2 = 10^{5+2} = 10^7$
2. $2^4 \cdot 2^5 = 2^{\_\_\_\_\_\_} = 2^{\_\_\_\_\_\_}$
3. $5^2 \cdot 5^6 = 5^{2+\_\_\_\_\_} = 5^{\_\_\_\_\_\_}$
4. $9^4 \cdot 9^6 = 9^{\_\_\_\_\_\_} = 9^{\_\_\_\_\_\_}$

34

Answer: 4+5, 9, 6, 8, 4+6, 10

Our first rule tells us that to find the products such as $a^x \cdot a^y$, we add the exponents.

$$a^x \cdot a^y = a^{x+y}$$

Therefore $2^2 \cdot 2^4 = 2^{2+4} = 2^6$

$6^5 \cdot 6^3 = 6^{5+3} =$ ________

35

Answer: 6^8

$6^{15} \cdot 6^{10} = 6^{25}$

$7^{14} \cdot 7^{21} =$ ________

36

Answer: 7^{35}

$a^3 \cdot a^5 = a^8$

$a^{21} \cdot a^5 =$ ________

37

Answer: a^{26}

$x^{25} \cdot x^{20} =$ ________

38

Answer: x^{45}

$2^a \cdot 2^b =$ ________

39

Answer: 2^{a+b}

Choose the correct answer for the following problem and place the corresponding letter in the parenthesis.

$2^3 \cdot 2^2 =$ ______ ( )

a. 2^6

b. 4^5

c. 4^6

d. 2^5

e. None of these

40

Answer: d (Remember $a^x \cdot a^y = a^{x+y}$. Therefore $2^3 \cdot 2^2 = 2^{3+2} = 2^5$.)

$5^{22} \cdot 5^{18} =$ ______

41

Answer: 5^{40}

$4^{25} \cdot 4^{30} = 4^{\_\_\_\_\_\_}$

42

Answer: 55

Choose the correct answer for the following problem and place the corresponding letter in the parenthesis.

$x^4 \cdot x^7 =$ ________ ( )

a. x^{28}

b. x^{47}

c. x^{11}

d. x^{22}

e. None of these

43

Answer: c (Since $x^4 \cdot x^7 = x^{4+7} = x^{11}$)

$x^{32} \cdot x^{15} = x^{\_\_\_\_}$

44

Answer: 47

$7^{57} \cdot 7^{63} = 7^{\_\_\_\_}$

45

Answer: 120

$a^1 \cdot a^5 = a^{\_\_\_\_}$

46

Answer: 6

Does $2^4 \cdot 2^3 = 2^{12}$? (yes or no) ________

47

Answer: No. Remember $a^x \cdot a^y = a^{x+y}$. Therefore $2^4 \cdot 2^3 = 2^{4+3} = 2^7$.

$37^{500} \cdot 37^{100} = 37^{\_\_\_\_\_\_}$

48

Answer: 600

$x^{1000} \cdot x^{750} = x^{\_\_\_\_\_\_}$

49

Answer: 1750

Choose the correct answer for the following problem and place the corresponding letter in the parenthesis.

$4^6 \cdot 4^5 =$ ______ ( )

a. 16^{11}

b. 16^{30}

c. 4^{11}

d. 4^{30}

e. None of these

50

Answer: c (Remember $a^x \cdot a^y = a^{x+y}$. Then $4^5 \cdot 4^6 = 4^{5+6} = 4^{11}$.)

$5^{500} \cdot 5^{1000} = 5^{\_\_\_\_\_\_}$

51

Answer: 1500

$x^{60} \cdot x^{120} = x^{\_\_\_\_}$

52

Answer: 180

$10^{16} \cdot 10^{13} = 10^{\_\_\_\_}$

53

Answer: 29

Does $5^{20} \cdot 5^{30} = 5^{600}$? (yes or no) ________

54

Answer: No. Remember $5^{20} \cdot 5^{30} = 5^{20+30} = 5^{50}$.

$x^{75} \cdot x^{150} = x^{\_\_\_\_}$

55

Answer: 225

$9^{5} \cdot 9^{3} = 9^{\_\_\_\_}$

$10^{16} \cdot 10^{14} = 10^{\_\_\_\_}$

$2^{28} \cdot 2^{64} = 2^{\_\_\_\_}$

$x^{600} \cdot x^{700} = x^{\_\_\_\_}$

56

Answer: 8, 30, 92, 1300

1. $3^4 \cdot 3^3$ = ________

2. $23^{16} \cdot 23^2$ = ________

3. $5^2 \cdot 5^8$ = ________

57

Answer: 1. 3^7, 2. 23^{18}, 3. 5^{10}

1. $2^a \cdot 2^b$ = ________

2. $x^{22} \cdot x^{18}$ = ________

3. $x^{30} \cdot x^{50}$ = ________

58

Answer: 1. 2^{a+b}, 2. x^{40}, 3. x^{80}

Rule 2: For each natural number a, and for all natural numbers x and y, $(a^x)^y = a^{x \cdot y} = a^{xy}$.

This rule tells us that in expressions such as $(a^x)^y$ we multiply the exponents.

$$(a^x)^y = a^{xy}$$

$$(a^2)^3 = a^{2 \cdot 3} = a^6$$

$$(2^2)^3 = 2^{2 \cdot 3} = 2^6$$

$$(4^5)^6 = 4^{5 \cdot 6} = 4^{30}$$

$$(5^3)^4 = 5^{3 \cdot 4} = \underline{\qquad\qquad}$$

59

Answer: 5^{12}

$(2^5)^7 = 2^{5 \cdot 7} = 2^{35}$

$(3^4)^5 =$ ________ $=$ ________

60

Answer: 3^{20} or $3^{4 \cdot 5}$

$(6^2)^3 = 6^6$

$(x^6)^4 =$ ________

61

Answer: x^{24}

Now let us see why rule 2 works.

$4^2 = 4 \cdot 4$

$4^3 = 4 \cdot 4 \cdot 4$

Therefore $(4^2)^3 = (4 \cdot 4) \cdot (4 \cdot 4) \cdot (4 \cdot 4) = 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 4^{\_\_\_}$

62

Answer: 6

In the expression $(4^2)^3 = (4 \cdot 4) \cdot (4 \cdot 4) \cdot (4 \cdot 4) = 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$, the 4 appears as a factor ________ times. Written in simpler form $(4^2)^3 = 4^{\_\_\_ \cdot \_\_\_} = 4^{\_\_\_}$.

63

Answer: 6, 2, 3, 6

Consider the expression $(5^3)^4$.

$5^3 = 5 \cdot 5 \cdot 5$

$5^4 = 5 \cdot 5 \cdot 5 \cdot 5$

Therefore $(5^3)^4 = (5 \cdot 5 \cdot 5) \cdot (5 \cdot 5 \cdot 5) \cdot (5 \cdot 5 \cdot 5) \cdot (5 \cdot 5 \cdot 5) =$

$5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5$

5 is used as a factor ________ times.

64

Answer: 12

By Rule 2, $(5^3)^4 = 5^{\_\_\_\_ \cdot \_\_\_\_} = 5^{\_\_\_\_}$

65

Answer: 3, 4, 12

1. $(9^5)^2 = 9^{5 \cdot 2} = 9^{10}$
2. $(2^4)^5 = 2^{\_\_\_\_} = 2^{\_\_\_\_}$
3. $(5^2)^6 = 5^{\_\_\_\_} = 5^{\_\_\_\_}$
4. $(10^4)^6 = 10^{\_\_\_\_} = 10^{\_\_\_\_}$

66

Answer: 2. $4 \cdot 5$, 20, 3. $2 \cdot 6$, 12, 4. $4 \cdot 6$, 24

Rule 2 tells that in expressions such as $(a^x)^y$ we multiply the exponents. $(a^x)^y = a^{xy}$

Therefore $(2^2)^4 = (2 \cdot 2) \cdot (2 \cdot 2) \cdot (2 \cdot 2) \cdot (2 \cdot 2) = 2^8$

$(6^5)^3 = (6 \cdot 6 \cdot 6 \cdot 6 \cdot 6) \cdot (6 \cdot 6 \cdot 6 \cdot 6 \cdot 6) \cdot (6 \cdot 6 \cdot 6 \cdot 6 \cdot 6)$

Therefore $(6^5)^3 =$ __________.

67

Answer: 6^{15}

$(6^8)^2 = 6^{16}$

$(7^9)^2 =$ __________

68

Answer: 7^{18}

$(a^3)^5 = a^{15}$

$(a^{10})^2 =$ __________

69

Answer: a^{20}

$(x^7)^3 =$ __________

70

Answer: x^{21}

$(2^a)^b =$ __________

Answer: 2^{ab} 71

Choose the correct answer to the following problem and put the corresponding letter in the parenthesis.

$(2^3)^2 =$ ________( )

a. 2^5

b. 4^5

c. 4^6

d. 2^6

e. None of these

72

Answer: d Remember $(a^x)^y = a^{xy}$. Therefore $(2^3)^2 = 2^{3 \cdot 2} = 2^6$.

$(5^3)^6 =$ ________

73

Answer: 5^{18}

$(4^{10})^3 =$ ________

74

Answer: 4^{30}

Choose the correct answer to the following problem and put the corresponding letter in the parenthesis.

$(x^4)^7 =$ ________ ( )

a. x^{47}

b. x^{11}

c. x^{28}

d. x^{22}

e. None of these

75

Answer: c since $(x^4)^7 = x^{4 \cdot 7} = x^{28}$

$(x^8)^3 = x^{\_\_\_\_}$

76

Answer: 24

$(7^5)^6 = 7^{\_\_\_\_}$

77

Answer: 30

$(a^2)^5 = a^{\_\_\_\_}$

78

Answer: 10

Does $(2^4)^3 = 2^7$? (yes or no) ________

79

Answer: No Remember $(a^x)^y = a^{xy}$. Therefore $(2^4)^3 = 2^{4 \cdot 3} = 2^{12}$.

$(37^{10})^{20} = 37^{\_\_\_\_\_\_}$

80

Answer: 200

$(x^{1000})^5 = x^{\_\_\_\_\_\_}$

81

Answer: 5000

Choose the correct answer for the following problem and put the corresponding letter in the parenthesis.

$(4^6)^5 =$ ______ ( )

a. 16^{11}

b. 4^{30}

c. 16^{30}

d. 4^{11}

e. None of these

82

Answer: b Remember $(a^x)^y = a^{xy}$. Therefore $(4^6)^5 = 4^{6 \cdot 5} = 4^{30}$.

$(5^5)^{100} = 5^{\_\_\_\_\_\_}$

83

Answer: 500

$(x^{60})^2 = x^{\_\_\_\_}$

84

Answer: 120

$(10^2)^{16} = 10^{\_\_\_\_}$

85

Answer: 32

Does $(5^{30})^{20} = 5^{50}$? (yes or no) ________

86

Answer: No Remember $(5^{30})^{20} = 5^{30 \cdot 20} = 5^{600}$

$(x^{75})^2 = x^{\_\_\_\_}$

87

Answer: 150

1. $(9^5)^3 = 9^{\_\_\_\_}$
2. $(10^{16})^{30} = 10^{\_\_\_\_}$
3. $(2^{64})^{20} = 2^{\_\_\_\_}$
4. $(x^{600})^7 = x^{\_\_\_\_}$

88

Answer: 1. 15, 2. 480, 3. 1280, 4. 4200

1. $(3^5)^4$ = ________
2. $(23^5)^{20}$ = ________
3. $(5^2)^8$ = ________
4. $(2^a)^b$ = ________
5. $(x^4)^{25}$ = ________
6. $(x^{30})^{50}$ = ________

89

Answer: 1. 3^{20} 2. 23^{100} 3. 5^{16} 4. 2^{ab} 5. x^{100}
6. x^{1500}

Rule 1: For each natural number a and for all natural numbers x and y, $a^x \cdot a^y = a^{x+y}$.

For example $3^2 \cdot 3^4 = 3^{2+4} = 3^6$.

Rule 2: For each natural number a and for every natural number x and y, $(a^x)^y = a^{xy}$.

For example $(3^2)^4 = 3^{2\cdot 4} = 3^8$.

1. $5^2 \cdot 5^6 = 5^{\_\_\_\_}$
2. $4^{50} \cdot 4^{25} = 4^{\_\_\_\_}$
3. $(x^{50})^4 = x^{\_\_\_\_}$
4. $100^4 \cdot 100^{24} = 100^{\_\_\_\_}$
5. $(3^{22})^3 = 3^{\_\_\_\_}$
6. $(7^{70})^{20} = 7^{\_\_\_\_}$
7. $x^{75} \cdot x^{25} = x^{\_\_\_\_}$

90

Answer: 1. 8 2. 75 3. 200 4. 28 5. 66

6. 1400 7. 100

1. $8^{16} \cdot 8^{32}$ = __________ 6. $x^{69} \cdot x^{31}$ = __________

2. $65^{10} \cdot 65^{30}$ = __________ 7. $(58^4)^{16}$ = __________

3. $(9^{25})^5$ = __________ 8. $19^{18} \cdot 19^2$ = __________

4. $x^{1500} \cdot x^{300}$ = __________ 9. $(x^{15})^{30}$ = __________

5. $(16^{10})^{15}$ = __________ 10. $(x^{100})^4$ = __________

91

Answer: 1. 8^{48} 2. 65^{40} 3. 9^{125} 4. x^{1800}

5. 16^{150} 6. x^{100} 7. 58^{64} 8. 19^{20}

9. x^{450} 10. x^{400}

DISCOVERY PROGRAM[21]

24

Answer: a^n

How many times is 5 used as a factor in this expression: $5\cdot5\cdot5\cdot5\cdot5\cdot5\cdot5$? ________ Written in simpler form $5\cdot5\cdot5\cdot5\cdot5\cdot5\cdot5 =$ ________.

25

Answer: 7, 5^7

In the expression $5^3 = 5\cdot5\cdot5$, the 5 is used as a factor ________ times.

In the expression $5^4 = 5\cdot5\cdot5\cdot5$, the 5 is used as a factor ________ times.

26

Answer: 3, 4

The product of $5^3\cdot5^4 = (5\cdot5\cdot5)\cdot(5\cdot5\cdot5\cdot5) = 5\cdot5\cdot5\cdot5\cdot5\cdot5\cdot5$. Five is used as a factor ________ times.

27

Answer: 7

In the product $5^3\cdot5^4 = (5\cdot5\cdot5)\cdot(5\cdot5\cdot5\cdot5)$, 5 is used as a factor 7 times. Written in simpler form $5^3\cdot5^4 =$ $(5\cdot5\cdot5)\cdot(5\cdot5\cdot5\cdot5) =$ ________.

---

[21]The first 23 frames of the Discovery Program are identical to the first 23 frames of the Traditional Program. See pp. 29-34.

Answer: 5^7 28

$4^2 = 4 \cdot 4$

$4^3 = 4 \cdot 4 \cdot 4$

Therefore $4^2 . 4^3 = (4 \cdot 4) \cdot (4 \cdot 4 \cdot 4) = 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 4^{\_\_\_\_}$.

Answer: 5 29

$4^5 . 4^2 = 4^{\_\_\_\_}$

Remember $4^5 = 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$

$4^2 = 4 \cdot 4$

Therefore $4^5 . 4^2 = (4 \cdot 4 \cdot 4 \cdot 4 \cdot 4) \cdot (4 \cdot 4) = 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$

30

Answer: 7

$5^1 . 5^2 = 5^{\_\_\_\_}$

Remember $5^1 = 5$, $5^2 = 5 \cdot 5$, therefore $5^1 . 5^2 = (5) \cdot (5 \cdot 5)$.

$6^3 . 6^4 = 6^{\_\_\_\_}$

31

Answer: 3, 7

$6^3 . 6^5 = 6^{\_\_\_\_}$

Remember $6^3 = 6 \cdot 6 \cdot 6$ and $6^5 = 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6$

$6^3 . 6^5 = (6 \cdot 6 \cdot 6) \cdot (6 \cdot 6 \cdot 6 \cdot 6 \cdot 6)$

$10^5 . 10^2 = 10^{\_\_\_\_}$

$2^4 . 2^5 = 2^{\_\_\_\_}$

32

Answer: 8, 7, 9

$8^3 \cdot 8^7 = 8^{\_\_\_\_}$

$5^2 \cdot 5^8 = 5^{\_\_\_\_}$

$10^6 \cdot 10^2 = 10^{\_\_\_\_}$

33

Answer: 10, 10, 8

Consider the problem $x^3 \cdot x^4$.

$x^3 = x \cdot x \cdot x$

x appears as a factor ________ times.

$x^4 = x \cdot x \cdot x \cdot x$

x appears as a factor ________ times.

$x^3 \cdot x^4 = (x \cdot x \cdot x) \cdot (x \cdot x \cdot x \cdot x) = x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x$

In the product, x appears as a factor ________ times.

Therefore $x^3 \cdot x^4 = x^7$.

34

Answer: 3, 4, 7

$x^2 \cdot x^5 = x^{\_\_\_\_}$

35

Answer: 7 Remember $x^2 = x \cdot x$, $x^5 = x \cdot x \cdot x \cdot x \cdot x$

$x^2 \cdot x^5 = (x \cdot x) \cdot (x \cdot x \cdot x \cdot x \cdot x) = x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x$

$x^3 \cdot x^5 = x^{\_\_\_\_}$

36

Answer: 8

Choose the correct answer for the following problem and place the corresponding letter in the parenthesis.

$2^3 \cdot 2^2 =$ ________ ( )

a. 2^6

b. 4^5

c. 4^6

d. 2^5

e. None of these

37

Answer: d (Remember $2^3 = 2 \cdot 2 \cdot 2$, $2^2 = 2 \cdot 2$, and $2^3 \cdot 2^2 = (2 \cdot 2 \cdot 2) \cdot (2 \cdot 2) = 2^5$)

$5^3 \cdot 5^4 = 5^{\_\_\_\_}$

38

Answer: 7

You have been working with products involving exponents. It is hoped that you have discovered something about finding these products. Without using paper or pencil choose the correct answer for the following problem and put the answer in the parenthesis.

$6^{15} \cdot 6^{10} =$ ________( )

a. 6^{25} b. 6^{150} c. Don't know

39

Answer: If you answered a go to page (frame) 45.

If you answered b go to page (frame) 39.

If you answered c go to page (frame) 40.

$6^{15}.6^{10}$ does <u>not</u> equal 6^{150}.

If you think $6^{15}.6^{10} = 6^{150}$ because $15\cdot10 = 150$ consider this problem: $2^3.2^4$

$2^3 = 2\cdot2\cdot2$

$2^4 = 2\cdot2\cdot2\cdot2$

$2^3.2^4 = (2\cdot2\cdot2)\cdot(2\cdot2\cdot2\cdot2) = 2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2 = 2^7$

$4\cdot3 = 12$ but $2^{12} = (2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2)$

Now go to the next page.

40

You have not discovered a shortcut for working problems like $6^{15}.6^{10}$.

Consider the problem $2^4.2^2$.

$2^4 = 2\cdot2\cdot2\cdot2$

$2^2 = 2\cdot2$

Therefore $2^4.2^2 = (2\cdot2\cdot2\cdot2)\cdot(2\cdot2) = 2\cdot2\cdot2\cdot2\cdot2\cdot2 = 2^{\_\_\_\_}$.

Answer: 6 41

$x^5 \cdot x^3 = (x \cdot x \cdot x \cdot x \cdot x) \cdot (x \cdot x \cdot x) = (x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x) = x^{\_\_\_}$

Therefore $x^5 \cdot x^3 = x^{\_\_\_}$.

42

Answer: 8, 8

$8^3 \cdot 8^4 = (8 \cdot 8 \cdot 8) \cdot (8 \cdot 8 \cdot 8 \cdot 8) = 8^{\_\_\_}$

43

Answer: 7

$7^2 \cdot 7^7 = (7 \cdot 7) \cdot (7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7) = 7^{\_\_\_}$

44

Answer: 9

$x^5 \cdot x^6 = x^{\_\_\_}$

Turn to page (frame) 51.

45

If you chose a you are correct and may therefore have found a shortcut. To check here are some more examples for you to work. If you just guessed turn back to page (frame) 40.

1. $6^5 \cdot 6^4 = 6^{\_\_\_}$
2. $2^{25} \cdot 2^5 = 2^{\_\_\_}$
3. $5^{38} \cdot 5^{62} = 5^{\_\_\_}$
4. $x^3 \cdot x^{10} = x^{\_\_\_}$
5. $x^{120} \cdot x^{100} = x^{\_\_\_}$

46

Answer: 1. 9, 2. 30, 3. 100, 4. 13, 5. 220

If you missed any of these and do not understand why turn back to page (frame) 40. Otherwise continue with page (frame) 46.

For each question in column I choose the correct answer from column II and place the corresponding letter in the parenthesis.

I		II
1. $2^5 \cdot 2^3$ = ______	( )	a. 8^{20}
2. $4^2 \cdot 4^6$ = ______	( )	b. 8^{12}
3. $8^{10} \cdot 8^2$ = ______	( )	c. 4^8
4. $2^7 \cdot 2^8$ = ______	( )	d. 2^8
5. $4^6 \cdot 4^6$ = ______	( )	e. 4^{12}
		f. 2^{15}
		g. None of these

47

Answer: 1. d, 2. c, 3. b, 4. f, 5. e

If you missed any of these write the products out in factored form to assure the correctness of the answer.

For example, $2^5 \cdot 2^3 = (2 \cdot 2 \cdot 2 \cdot 2 \cdot 2) \cdot (2 \cdot 2 \cdot 2) = 2^8$

$9^{13} \cdot 9^{12} = 9^{\_\_\_\_}$

$10^{28} \cdot 10^{38} = 10^{\_\_\_\_}$

$x^{50} \cdot x^{40} = x^{\_\_\_\_}$

48

Answer: 25, 60, 90

If you missed any of these and do not understand why go back to page (frame) 40. Otherwise continue with page (frame) 48.

Choose the correct answer to the following problem and put the corresponding letter in the parenthesis.

$$3^2 \cdot 3^5 = \_\_\_\_\_\_ \quad (\quad)$$

a. 3^7

b. 9^{10}

c. 9^7

d. 3^{10}

e. None of these

49

Answer: a, since $3^2 = 3 \cdot 3$, and $3^5 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$. Therefore

$$3^2 \cdot 3^5 = (3 \cdot 3) \cdot (3 \cdot 3 \cdot 3 \cdot 3 \cdot 3) = 3^7.$$

1. $3^7 \cdot 3^8 = 3^{\_\_\_\_}$
2. $4^{25} \cdot 4^{30} = 4^{\_\_\_\_}$
3. $22^{16} \cdot 22^8 = 22^{\_\_\_\_}$
4. $59^{23} \cdot 59^{68} = 59^{\_\_\_\_}$
5. $x^{500} \cdot x^{800} = x^{\_\_\_\_}$

50

Answer: 1. 15, 2. 55, 3. 24, 4. 91, 5. 1300

Turn now to page (frame) 59 if you got these correct.

Otherwise turn back to page (frame) 40.

51

Answer: From page (frame) 44 - 11

It is now hoped that you have found a shortcut for working problems like $5^{20} \cdot 5^{30}$. Without paper or pencil choose the correct answer for the following problem and put the corresponding letter in the parenthesis.

$5^{20} \cdot 5^{30}$ = ________ ( )

a. 5^{50}

b. 5^{600}

c. Do not know

52

Answer: If you answered a turn to page (frame) 55.

If you answered b turn to page (frame) 52.

If you answered c turn to page (frame) 53.

$5^{20} \cdot 5^{30}$ does <u>not</u> equal 5^{600}.

If you think $5^{20} \cdot 5^{30} = 5^{600}$ because $20 \cdot 30 = 600$ consider this problem: $3^2 \cdot 3^3$

$3^2 = 3 \cdot 3$

$3^3 = 3 \cdot 3 \cdot 3$

$3^2 \cdot 3^3 = (3 \cdot 3) \cdot (3 \cdot 3 \cdot 3) = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 3^5$

$3^6 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

Now turn to page (frame) 53.

53

You have not yet discovered a shortcut for solving problems like $5^{20} \cdot 5^{30}$.

Consider the simpler problem $2^2 \cdot 2^4$.

$2^2 \cdot 2^4 = (2 \cdot 2) \cdot (2 \cdot 2 \cdot 2 \cdot 2) = 2^{\_\_\_}$

54

Answer: 6

1. $9^5 \cdot 9^2 = 9^{\_\_\_}$

2. $x^8 \cdot x^4 = (x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x) \cdot (x \cdot x \cdot x \cdot x)$

   $x^8 \cdot x^4 = x^{8+4} = x^{\_\_\_}$

3. $15^{32} \cdot 15^{45} = 15^{\_\_\_ + \_\_\_} = 15^{\_\_\_}$

Turn to page (frame) 59.

55

You got the problem correct and may therefore have found a shortcut for solving problems like $5^{20} \cdot 5^{30}$. To check here are some more problems to work. If you just guessed turn back to page (frame) 53.

1. $4^{13} \cdot 4^{28} = 4^{\_\_\_\_}$
2. $10^{500} \cdot 10^{60} = 10^{\_\_\_\_}$
3. $x^{82} \cdot x^{58} = x^{\_\_\_\_}$

56

Answer: 1. 41, 2. 560, 3. 140

If you missed any of these and do not understand why turn back to page (frame) 53. Otherwise continue with page (frame) 56.

Choose the correct answer for the following problem and place the corresponding letter in the parenthesis.

$5^4 \cdot 5^5 =$ ______ ( )

a. 5^{20}

b. 5^9

c. 25^{20}

d. 25^9

e. None of these

57

Answer: b $5^4 \cdot 5^5 = (5\cdot5\cdot5\cdot5)\cdot(5\cdot5\cdot5\cdot5\cdot5) = 5^9$

1. $10^{16} \cdot 10^{13} = 10^{\_\_\_\_}$
2. $9^{25} \cdot 9^{75} = 9^{\_\_\_\_}$
3. $37^{500} \cdot 37^{100} = 37^{\_\_\_\_}$
4. $x^{1000} \cdot x^{750} = x^{\_\_\_\_}$

58

Answer: 1. 29, 2. 100, 3. 600, 4. 1750

Turn now to page (frame) 59.

59

Answer: 1. 7, 2. 12, 3. 32, 45, 77 (From page (frame) 54)

1. $9^5 \cdot 9^3 = 9^{\_\_\_\_}$
2. $10^{16} \cdot 10^{14} = 10^{\_\_\_\_}$
3. $2^{78} \cdot 2^{64} = 2^{\_\_\_\_}$
4. $x^{600} \cdot x^{700} = x^{\_\_\_\_}$

60

Answer: 1. 8, 2. 30, 3. 142, 4. 1300

1. $3^4 \cdot 3^3 =$ ______
2. $23^{16} \cdot 23^{23} =$ ______
3. $5^2 \cdot 5^8 =$ ______
4. $2^a \cdot 2^b =$ ______
5. $x^{27} \cdot x^{18} =$ ______
6. $x^{30} \cdot x^{50} =$ ______

61

Answer: 1. 3^7, 2. 23^{39}, 3. 5^{10}, 4. 2^{a+b}, 5. x^{45}, 6. x^{80}

How many times is 4 used as a factor in this expression: $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$? ________ Written in simpler form $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 =$ __________.

62

Answer: 6, 4^6

$4^2 = 4 \cdot 4$: In this expression 4 is used as a factor ________ times.

In the expression $(4^2)^3 = 4^2 \cdot 4^2 \cdot 4^2$, the 4^2 is used as a factor ________ times.

63

Answer: 2, 3

Since $4^2 = 4 \cdot 4$, $(4^2)^3 = (4^2) \cdot (4^2) \cdot (4^2) = (4 \cdot 4) \cdot (4 \cdot 4) \cdot (4 \cdot 4)$.

The 4 is used as a factor ________ times.

64

Answer: 6

In the expression $(4^2)^3 = (4^2) \cdot (4^2) \cdot (4^2) = (4 \cdot 4) \cdot (4 \cdot 4) \cdot (4 \cdot 4)$, the 4 is used as a factor 6 times. Written in simpler form $(4^2)^3 =$ ________.

65

Answer: 4^6

In the expression $5^3 = 5 \cdot 5 \cdot 5$, the 5 is used as a factor ________ times.

In the expression $(5^3)^4 = 5^3 \cdot 5^3 \cdot 5^3 \cdot 5^3$, the 5^3 is used as a factor ________ times.

66

Answer: 3, 4

In the expression $(5^3)^4 = (5^3) \cdot (5^3) \cdot (5^3) \cdot (5^3) =$ $(5 \cdot 5 \cdot 5) \cdot (5 \cdot 5 \cdot 5) \cdot (5 \cdot 5 \cdot 5) \cdot (5 \cdot 5 \cdot 5)$, the 5 is used as a factor ________ times. A simpler way of writing $(5^3)^4$ is ________.

67

Answer: 12, 5^{12}

$(5^3)^4 = 5^{\_\_\_\_}$

Remember $(5^3)^4 = (5 \cdot 5 \cdot 5) \cdot (5 \cdot 5 \cdot 5) \cdot (5 \cdot 5 \cdot 5) \cdot (5 \cdot 5 \cdot 5)$

68

Answer: 12

$(5^1)^2 = 5^{\_\_\_\_}$ Remember $5^1 = 5$. Therefore $(5^1)^2 =$

$(5^1) \cdot (5^1) = 5 \cdot 5$.

$(6^3)^4 = 6^{\_\_\_\_}$

69

Answer: 2, 12

$(6^3)^5 = 6^{\_\_\_\_}$ Remember $6^3 = 6 \cdot 6 \cdot 6$. $(6^3)^5 =$
$6^3 \cdot 6^3 \cdot 6^3 \cdot 6^3 \cdot 6^3 = (6 \cdot 6 \cdot 6) \cdot (6 \cdot 6 \cdot 6) \cdot (6 \cdot 6 \cdot 6) \cdot (6 \cdot 6 \cdot 6) \cdot (6 \cdot 6 \cdot 6)$

$(10^5)^2 = 10^{\_\_\_\_}$

$(2^4)^5 = 2^{\_\_\_\_}$

70

Answer: 15, 10, 20

$(8^3)^7 = 8^{\_\_\_\_}$

$(5^2)^6 = 5^{\_\_\_\_}$

$(10^4)^6 = 10^{\_\_\_\_}$

71

Answer: 21, 12, 24

Consider the problem $(x^3)^4$.

$x^3 = x \cdot x \cdot x$

x appears as a factor ______ times.

$(x^3)^4 = x^3 \cdot x^3 \cdot x^3 \cdot x^3$

x^3 appears as a factor ______ times.

$(x^3)^4 = (x^3) \cdot (x^3) \cdot (x^3) \cdot (x^3) = (x \cdot x \cdot x) \cdot (x \cdot x \cdot x) \cdot (x \cdot x \cdot x) \cdot (x \cdot x \cdot x)$

x appears as a factor ______ times.

Therefore $(x^3)^4 = x^{\_\_\_\_}$.

Answer: 3, 4, 12, 12 72

$(x^2)^5 = x^{\_\_\_\_}$

Answer: 10 Remember $(x^2)^5 = x^2 \cdot x^2 \cdot x^2 \cdot x^2 \cdot x^2$, but $x^2 = x \cdot x$. 73

Therefore $(x^2)^5 = (x \cdot x) \cdot (x \cdot x) \cdot (x \cdot x) \cdot (x \cdot x) \cdot (x \cdot x)$.

$(x^3)^5 = x^{\_\_\_\_}$

74

Answer: 15

Choose the correct answer to the following problem and place the corresponding letter in the parenthesis.

$(2^3)^2 = \_\_\_\_\_\_$ ( )

a. 4^6

b. 2^6

c. 4^5

d. 2^5

e. None of these

75

Answer: b, since $(2^3)^2 = 2^3 \cdot 2^3 = (2 \cdot 2 \cdot 2) \cdot (2 \cdot 2 \cdot 2) = 2^6$.

$(5^3)^4 = 5^{\_\_\_\_}$

76

Answer: 12

You have been working problems like $(5^3)^4$ and it is hoped that you have discovered something about solving such problems. Without paper or pencil, choose the correct answer to the following problem and place the corresponding letter in the parenthesis.

$(6^{15})^{10}$ = ______ ( )

a. 6^{150}

b. 6^{25}

c. do not know

77

Answer: If you answered a turn to page (frame) 82.

If you answered b turn to page (frame) 77.

If you answered c turn to page (frame) 78.

$(6^{15})^{10}$ does <u>not</u> equal 6^{25}.

If you think $(6^{15})^{10} = 6^{25}$ because $15 + 10 = 25$ consider this problem: $(2^3)^4 = 2^3 \cdot 2^3 \cdot 2^3 \cdot 2^3 =$

$(2\cdot2\cdot2)\cdot(2\cdot2\cdot2)\cdot(2\cdot2\cdot2)\cdot(2\cdot2\cdot2) = 2^{12}$.

$3 + 4 = 7$ but $2^7 = 2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2$.

Now turn to page (frame) 78.

78

You have not discovered a shortcut for working problems like $(6^{15})^{10}$. Consider the simpler problem $(2^4)^2$.

$(2^4)^2 = (2^4)\cdot(2^4) = (2\cdot2\cdot2\cdot2)\cdot(2\cdot2\cdot2\cdot2) = 2^{\underline{\quad\quad}}$

79

Answer: 8

$(x^5)^3 = x^5\cdot x^5\cdot x^5 = (x\cdot x\cdot x\cdot x\cdot x)\cdot(x\cdot x\cdot x\cdot x\cdot x)\cdot(x\cdot x\cdot x\cdot x\cdot x)$

$= x^{\underline{\quad\quad}}$

Therefore $(x^5)^3 = x^{\underline{\quad\quad}}$.

80

Answer: 15, 15

$(8^3)^4 = 8^3\cdot8^3\cdot8^3\cdot8^3 = (8\cdot8\cdot8)\cdot(8\cdot8\cdot8)\cdot(8\cdot8\cdot8)\cdot(8\cdot8\cdot8)$

$= 8^{\underline{\quad\quad}}$

81

Answer: 12

$(x^5)^6 = x^{\underline{\quad\quad}}$

Turn now to page (frame) 88.

82

If you chose a, you are correct and may therefore have found a shortcut. To check, here are some more problems for you to work. If you just guessed, turn now back to page (frame) 78.

1. $(6^5)^4 = 6^{\_\_\_\_}$
2. $(2^{10})^5 = 2^{\_\_\_\_}$
3. $(5^{15})^2 = 5^{\_\_\_\_}$
4. $(x^6)^2 = x^{\_\_\_\_}$
5. $(x^3)^{10} = x^{\_\_\_\_}$

83

Answer: 1. 20, 2. 50, 3. 30, 4. 12, 5. 30

If you missed any of these and do not understand why, turn back to page (frame) 78. Otherwise continue with page (frame) 83.

For each question in column I choose the correct answer from column II and place the corresponding letter in the parenthesis.

I.		II.
1. $(2^5)^2$ = ______	( )	a. 2^{10}
2. $(4^2)^6$ = ______	( )	b. 2^8
3. $(8^{10})^4$ = ______	( )	c. 8^{12}
4. $(2^2)^4$ = ______	( )	d. 4^{36}
5. $(4^6)^6$ = ______	( )	e. 8^{40}
		f. 4^{12}

84

Answer: 1. a, 2. f, 3. e, 4. b, 5. d

If you missed any of these, write the expression out in factored form to assure the correctness of the answer.

For example: $(2^5)^2 = 2^5 \cdot 2^5 = (2\cdot2\cdot2\cdot2\cdot2)\cdot(2\cdot2\cdot2\cdot2\cdot2) = 2^{10}$

$(9^{13})^3 = 9^{\_\_\_\_}$

$(10^4)^8 = 10^{\_\_\_\_}$

$(x^{50})^4 = x^{\_\_\_\_}$

85

Answer: 39, 32, 200

If you missed any of these and do not understand why, turn now to page (frame) 78. Otherwise continue with page (frame) 85.

Choose the correct answer for the following problem and place the corresponding letter in the parenthesis.

$(3^2)^5 =$ ________ ( )

a. 3^7

b. 9^{10}

c. 9^7

d. 3^{10}

e. None of these

86

Answer: d, since $(3^2)^5 = 3^2 \cdot 3^2 \cdot 3^2 \cdot 3^2 \cdot 3^2$ and $3^2 = 3 \cdot 3$.

Therefore $(3^2)^5 = (3 \cdot 3) \cdot (3 \cdot 3) \cdot (3 \cdot 3) \cdot (3 \cdot 3) \cdot (3 \cdot 3)$
$= 3^{10}$.

1. $(3^7)^8 = 3^{\_\_\_\_}$
2. $(4^{25})^3 = 4^{\_\_\_\_}$
3. $(22^{16})^2 = 22^{\_\_\_\_}$
4. $(59^3)^9 = 59^{\_\_\_\_}$
5. $(x^{500})^8 = x^{\_\_\_\_}$

87

Answer: 1. 56, 2. 75, 3. 32, 4. 27, 5. 4000

If you answered all these correctly, turn now to page (frame) 96. Otherwise turn back to page (frame) 78.

88

Answer: 30 - $(x^5)^6 = x^5 \cdot x^5 \cdot x^5 \cdot x^5 \cdot x^5 \cdot x^5 = x^{30}$ since

$x^5 = x \cdot x \cdot x \cdot x \cdot x$ (Answer from page (frame) 81)

It is now hoped that you have discovered a shortcut for working problems like $(x^5)^6$. Without paper or pencil, choose the correct answer for the following problem and put the corresponding letter in the parenthesis.

$(5^{20})^{30} =$ ______ ( )

a. 5^{600}

b. 5^{50}

c. Do not know

89

Answer: If you answered a turn to page (frame) 92.

If you answered b turn to page (frame) 89.

If you answered c turn to page (frame) 90.

$(5^{20})^{30}$ does <u>not</u> equal 5^{50}.

If you think $(5^{20})^{30} = 5^{50}$ because $20 + 30 = 50$ consider this problem: $(3^2)^3 = 3^2 \cdot 3^2 \cdot 3^2 = (3\cdot3)\cdot(3\cdot3)\cdot(3\cdot3) = 3^6$.

$2 + 3$ does not equal 6. $3^5 = 3\cdot3\cdot3\cdot3\cdot3$

Now turn to page (frame) 90.

90

You have not yet found a shortcut for working problems like $(5^{20})^{30}$.

Consider the simpler problem $(2^2)^4$.

$(2^2)^4 = 2^2 \cdot 2^2 \cdot 2^2 \cdot 2^2 = (2\cdot2)\cdot(2\cdot2)\cdot(2\cdot2)\cdot(2\cdot2) = 2^{\_\_\_\_}$

91

Answer: 8

1. $(9^5)^2 = 9^{\_\_\_\_}$
2. $(x^8)^2 = (x\cdot x\cdot x\cdot x\cdot x\cdot x\cdot x\cdot x)\cdot(x\cdot x\cdot x\cdot x\cdot x\cdot x\cdot x\cdot x)$

   $(x^8)^2 = x^{8\cdot2} = x^{\_\_\_\_}$
3. $(15^3)^{15} = 15^{\_\_\_\_\cdot\_\_\_\_} = 15^{\_\_\_\_}$

Turn now to page (frame) 96.

92

You got the problem correct and may therefore have found a shortcut. To check, here are some more problems for you to work. If you just guessed, turn now back to page (frame) 90.

1. $(4^{28})^2 = 4^{\_\_\_\_}$
2. $(10^{50})^6 = 10^{\_\_\_\_}$
3. $(x^{80})^3 = x^{\_\_\_\_}$

93

Answer: 1. 56, 2. 300, 3. 240

If you missed any of these and do not understand why, turn back to page (frame) 90. Otherwise continue with page (frame) 93.

Choose the correct answer to the following problem and place the corresponding letter in the parenthesis.

$(5^4)^5 =$ ______ ( )

a. 5^9

b. 5^{20}

c. 25^{20}

d. 25^9

e. None of these

94

Answer: b - $(5^4)^5 = 5^4 \cdot 5^4 \cdot 5^4 \cdot 5^4 \cdot 5^4 =$

$(5\cdot5\cdot5\cdot5)\cdot(5\cdot5\cdot5\cdot5)\cdot(5\cdot5\cdot5\cdot5)\cdot(5\cdot5\cdot5\cdot5)\cdot(5\cdot5\cdot5\cdot5) = 5^{20}$

1. $(10^{14})^2 = 10^{\underline{\qquad}}$
2. $(9^{25})^4 = 9^{\underline{\qquad}}$
3. $(37^{10})^{20} = 37^{\underline{\qquad}}$
4. $(x^{10})^5 = x^{\underline{\qquad}}$

95

Answer: 1. 28, 2. 100, 3. 200, 4. 50

If you answered these correctly, turn now to page (frame) 96. Otherwise turn back to page (frame) 90.

96

Answer: 1. 10, 2. 16, 3. 3, 15, 45 (From page (frame) 91)

1. $(9^5)^3 = 9^{\underline{\qquad}}$
2. $(10^{16})^{30} = 10^{\underline{\qquad}}$
3. $(2^{64})^{20} = 2^{\underline{\qquad}}$
4. $(x^{600}) = x^{\underline{\qquad}}$

Answer: 1. 15, 2. 480, 3. 1280, 4. 4200

1. $(3^5)^4$ = ________
2. $(23^5)^{20}$ = ________
3. $(5^2)^8$ = ________
4. $(2^a)^b$ = ________
5. $(x^4)^{25}$ = ________
6. $(x^{30})^{50}$ = ________

Answer: 1. 3^{20}, 2. 23^{100}, 3. 5^{16}, 4. 2^{ab}, 5. x^{100}
6. x^{1500}

1. $5^2 \cdot 5^6 = 5^{\_\_\_\_}$
2. $4^{50} \cdot 4^{25} = 4^{\_\_\_\_}$
3. $(x^{50})^4 = x^{\_\_\_\_}$
4. $100^4 \cdot 100^{24} = 100^{\_\_\_\_}$
5. $(3^{22})^3 = 3^{\_\_\_\_}$
6. $(7^{70})^{20} = 7^{\_\_\_\_}$
7. $x^{75} \cdot x^{25} = x^{\_\_\_\_}$

99

Answer: 1. 8, 2. 75, 3. 200, 4. 28, 5. 66
6. 1400, 7. 100

1. $8^{16} \cdot 8^{32}$ = ________

2. $65^{10} \cdot 65^{30}$ = ________

3. $(9^{25})^5$ = ________

4. $x^{1500} \cdot x^{300}$ = ________

5. $(16^{10})^{15}$ = ________

6. $x^{69} \cdot x^{31}$ = ________

7. $(58^4)^{16}$ = ________

8. $19^{18} \cdot 19^2$ = ________

9. $(x^{15})^{30}$ = ________

10. $(x^{100})^4$ = ________

100

Answer: 1. 8^{48}, 2. 65^{40}, 3. 9^{125}, 4. x^{1800},
5. 16^{150}, 6. x^{100}, 7. 58^{64}, 8. 19^{20},
9. x^{450}, 10. x^{400}

APPENDIX C

APPENDIX C

PRE-TEST

Write the following problems in simpler form. Do not find the product.

1. $5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 =$ ________

2. $2^3 \cdot 2^4 =$ ________

3. $5^{10} \cdot 5^{20} =$ ________

4. $(3^2)^5 =$ ________

5. $(6^9)^{11} =$ ________

POST TEST

For each of the following problems, choose the correct answer from the choices given and place the corresponding letter in the parenthesis.

1. $2^3 \cdot 2^2$ = ______ ( )
   a. 4^6
   b. 4^5
   c. 2^6
   d. 2^5
   e. None of these

2. $(4^2)^3$ = ______ ( )
   a. 16^5
   b. 4^6
   c. 4^5
   d. 16^5
   e. None of these

3. $x^a \cdot x^b$ = ______ ( )
   a. x^{ab}
   b. $2x^{ab}$
   c. $2x^{a+b}$
   d. x^{a+b}
   e. None of these

4. $[(4^3)^8]^2$ = ______ ( )
   a. 4^{26}
   b. 4^{48}
   c. 4^{13}
   d. 16^{48}
   e. None of these

5. $11^9 \cdot 11^{20}$ = ______ ( )
   a. 11^{29}
   b. 11^{180}
   c. 121^{29}
   d. 121^{180}
   e. None of these

6. $(a^p)^q$ = ______ ( )
   a. pa^q
   b. pa^{pq}
   c. a^{p+q}
   d. a^{pq}
   e. None of these

7. $(7^4 \cdot 7^8)^2$ = ______ ( )

a. 49^{14}

b. 49^{24}

c. 7^{64}

d. 7^{14}

e. None of these

8. $10^a \cdot 10^b$ = ______ ( )

a. 100^{a+b}

b. 10^{a+b}

c. 100^{ab}

d. 10^{ab}

e. None of these

9. $6^4 \cdot 6^2 \cdot 6^1$ = ______ ( )

a. 216^7

b. 216^8

c. 6^7

d. 6^8

e. None of these

10. $(x^2)^3 \cdot (x^4)^5$ = ____( )

a. x^{120}

b. x^{45}

c. x^{26}

d. x^{14}

e. None of these

11. $[(p^2)^2 \cdot (p^4)^3]^2$ = ___ ( )

a. p^{13}

b. p^{18}

c. p^{96}

d. p^{16}

e. None of these

12. $[(2^3)^3 \cdot (2^2)^5]^2$ = ___ ( )

a. 2^{38}

b. 2^{21}

c. 2^{15}

d. 2^{180}

e. None of these

13. $(5^x)^y$ = ______ ( )

a. $x5^{xy}$

b. $x5^y$

c. 5^{xy}

d. 5^{x+y}

e. None of these

14. $(x^1 \cdot x^5)^{10}$ = ______ ( )

a. x^{16}

b. x^{15}

c. x^{60}

d. x^{50}

e. None of these

15. $(9^2)^a$ = ____________ ( )

a. 81^{2a}

b. 9^{2+a}

c. 81^{2+a}

d. 9^{2a}

e. None of these

16. $4^5 . 4^2 . 4^3$ = ________ ( )

a. 4^{30}

b. 4^{13}

c. 64^{30}

d. 64^{13}

e. None of these

17. $[(x^2)^6]^3$ = _________ ( )

a. x^{36}

b. x^{15}

c. x^{11}

d. x^{24}

e. None of these

18. $(6^4 . 6^3)^2$ = ________ ( )

a. 6^{14}

b. 6^9

c. 36^9

d. 36^{14}

e. None of these

19. $(7^1)^3 . (7^2)^4$ = ____ ( )

a. 7^{24}

b. 49^{11}

c. 7^{10}

d. 7^{11}

e. None of these

20. $[(a^1)^2 . (a^2)^4]^3$ = __ ( )

a. a^{12}

b. a^{30}

c. a^{13}

d. a^{48}

e. None of these

21. $(3^2)^4 . (3^1)^5$ = ____ ( )

a. 3^{40}

b. 9^{40}

c. 3^{30}

d. 3^{11}

e. None of these

22. $20^2 . 20^{10}$ = ________ ( )

a. 20^{20}

b. 400^{20}

c. 20^{12}

d. 400^{12}

e. None of these

23. $[(x^5)^{10}]^2$ = ______ ( )

a. x^{17}

b. x^{52}

c. x^{30}

d. x^{100}

e. None of these

24. $[2^2 \cdot 2^3 \cdot 2^4]^5$ = ____ ( )

a. 8^{45}

b. 8^{100}

c. 2^{45}

d. 2^{11}

e. None of these

25. $(9^2 \cdot 9^3)^4$ = ______ ( )

a. 81^9

b. 9^{20}

c. 81^{20}

d. 9^{24}

e. None of these

26. $12^x \cdot 12^y$ = ______ ( )

a. 144^{xy}

b. 144^{x+y}

c. 12^{x+y}

d. 12^{xy}

e. None of these

27. $(x^1 \cdot x^4 \cdot x^3)^2$ = ____ ( )

a. x^{16}

b. x^{24}

c. x^{14}

d. x^{10}

e. None of these

28. $[(2^2)^3]^1 \cdot [(2^4)^5]^2$ =__( )

a. 4^{46}

b. 2^{17}

c. 2^{46}

d. 4^{17}

e. None of these

29. $[(x^1)^2]^3 \cdot [(x^2)^3]^5$ =__( )

a. x^{36}

b. x^{16}

c. x^{60}

d. x^{180}

e. None of these

30. $(10^2)^5$ = ______ ( )

a. 10^{10}

b. 100^7

c. 10^7

d. 100^{10}

e. None of these