A STUDY OF PIAGET'S THEORY CONCERNING THE RELATIONSHIP BETWEEN DISTANCE CONSERVATION AND TWO-DIMENSIONAL AWARENESS

SANDRA ADAMS WALTON

A STUDY OF PIAGET'S THEORY CONCERNING THE RELATIONSHIP BETWEEN DISTANCE CONSERVATION AND TWO-DIMENSIONAL AWARENESS

An Abstract

Presented to

the Graduate Council of

Austin Peay State University

In Partial Fulfillment

of the Requirements for the Degree

Master of Arts

in Education

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by

Sandra Adams Walton

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ABSTRACT

The Swiss psychologist, Jean Piaget, has advanced a developmental theory of the cognitive development of the child in which he asserts that conservation of distance precedes the child's command of two-dimensional skills. Piaget uses a variety of tasks to determine the child's ability to conserve distance and his ability to manipulate within a two-dimensional environment. Of special interest is Piaget's division of two-dimensional skills into tests for understanding of first horizontality, then verticality.

The current study is devoted to determine if indeed distance conservation is a prerequisite for two-dimensional skills. This investigator differs from Piaget in his interpretation of two-dimensional skills. Instead of using separate tests for horizontality and verticality, this investigator has developed a model which, in his opinion, tests the child's command of a two-dimensional environment utilizing a purely mathematical interpretation. That is, the two-dimensional environment is perceived in terms of the Cartesian co-ordinate plane.

Sixty children enrolled in the second grade at Barksdale Elementary School, Clarksville, Tennessee, were individually interviewed by this investigator. Their ability to manipulate in a two-dimensional environment was first tested. Then the investigator attempted to identify those children who conserved distance.

According to the type of tests used here, in terms of the mathematical interpretation of two dimensions, the results of this study fail to reject the hypothesis that distance conservation is not a developmental prerequisite for two-dimensional skills.

A STUDY OF PIAGET'S THEORY CONCERNING THE RELATIONSHIP BETWEEN DISTANCE CONSERVATION AND TWO-DIMENSIONAL AWARENESS

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To the Graduate Council:

I am submitting herewith a Thesis written by Sandra Adams Walton entitled "A Study of Piaget's Theory Concerning the Relationship Between Distance Conservation and Two-Dimensional Awareness." I recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Arts in Education, with a major in mathematics.

Ernest Woodward
Major Professor

We have read this thesis and recommend its acceptance:

Elois B. Burns
Minor Professor

William G. Stokes
Third Committee Member

Accepted for the Council:

Wayne E. Stamper
Dean of the Graduate School

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CHAPTER I

INTRODUCTION

I. GENERAL STATEMENT OF THE PROBLEM AND REVIEW OF THE LITERATURE

The rationale for this study is perhaps best explained in the words of the Swiss psychologist, Jean Piaget. In the preface to The Child's Conception of Space, which Piaget coauthored with Barbel Inhelder, he states:

> Geometry primers are almost unanimous in presenting the fundamental ideas of space as resting upon euclidean concepts such as straight lines, angles, squares, circles, measurements, and the like...On the other hand, abstract geometrical analysis tends to show that fundamental spatial concepts are not euclidean at all, but 'topological'...It need hardly be said that a further reason for devoting special attention to the question of the development of spatial concepts, is that any reasonably thoroughgoing investigation in this field is certain to suggest immediate practical applications. The teaching of geometry could hardly fail to profit from keeping to the natural pattern of development of geometrical thought, especially since this process is, to our way of thinking, in much closer conformity with the logic of mathematical construction than are most of the so-called 'elementary' textbooks.[1]

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[1] Jean Piaget, and Barbel Inhelder, The Child's Conception of Space (New York: W. W. Norton and Company, Inc. 1967), p. vii.

Piaget advances a theory of the cognitive development of the child which, when applied in the area of mathematics education, has broad ramifications. He is saying that in the child's growing conception of space, topological concepts precede projective concepts, which in turn precede Euclidean concepts. Within these broad categories are numerous substages which Piaget contends must precede, and thus implement, a subsequent stage. Should this theory be found correct, the teacher and textbook author may have many changes to make in the sequencing of topics for presentation to the child.

Piaget states that

> As distinct from elementary topological relations which are concerned simply with the object as a thing in itself and with its various features taken in turn, we have shown (in preceding chapters) that projective concepts imply a comprehensive linking together of figures in a single system, based on the co-ordination of a number of different viewpoints. But side by side with the development of this organized complex of viewpoints there also takes place a co-ordination of objects as such. This leads ultimately to the idea of euclidean space, the concepts of parallels, angles and proportion providing the transition between the two systems. Such a co-ordination of objects naturally ASSUMES THE CONSERVATION OF DISTANCE (emphasis is that of the present author.)...[2]

The present study is designed to investigate an admittedly small portion of this rather extensive theory. Under consideration

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[2] Ibid., p. 375.

here is an analysis of Piaget's contention that the substage of distance conservation (later defined) has developmental priority over that of two-dimensional skills.

Richard W. Copeland describes the four main stages in Piaget's developmental theory as follows:

> Sensori-motor Stage. The first is a sensori-motor preverbal stage lasting approximately for the first two years of life.
>
> Pre-operational Stage. The second stage is that of pre-operational representation. It marks the beginning of language in the form of words...Words are symbols or representations of reality...This use of symbols or 'representation' marks the beginning of thought. At this stage there is a reconstruction or representation of the experiences at the pre-representational or sensorimotor level...
>
> Concrete Operations Stage. The third stage is that of concrete operations, which is particularly important to the elementary school teacher because most of the time that children are in the elementary school they are in this stage of development... This is a concrete operational stage because the child is obtaining ideas from operations on such concrete objects as water, clay, etc. At the beginning of the concrete operational level the ideas of the child are still based on observation and experience with objects in the physical world, but he is beginning to generalize or break away from manipulation of objects as a way of "knowing." When these generalizations are complete and correct the child is at the concrete-operational level...
>
> Formal Operation Stage. The last or fourth stage is the formal operation stage or the hypothetic-deductive operational level. This does not usually occur until eleven to twelve years of age. The child now reasons of

> hypotheses or ideas rather than on needing objects in the physical world as a basis for his thinking. He constructs new operations, those of propositional logic, and he attains new mental structures. He is now able to consider all possibilities or combinations rather than those based only on experience or experiment. He begins to classify, order, and enumerate in the verbal proposition form of deductive logic...[3]

Of interest here are the studies made pertaining to the Pre-operational and Concrete Stages with particular attention being given to the substages involved in the acquisition of two-dimensional skills.

Piaget emphasizes that the "simplest and most natural reference frame available to the child is most probably that provided by the physical world in the shape of vertical and horizontal axes...On the one hand, the concepts of vertical and horizontal are by nature physical rather than mathematical, indicating as they do, simply the direction taken by a freely falling body or a line perpendicular to it. Yet on the other hand, the elaboration of these concepts introduces a question independent of physics, or at any rate, a question whose independence is precisely the point at issue...and this is

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[3]Richard W. Copeland, <u>How Children Learn Mathematics</u> (New York: The Macmillan Company, 1970), pp. 14-16.

the development of a co-ordinate system as a simple tool of geometrical orientation."[4]

By asking children to fill in the water level on pictures of variously shaped and frequently inverted jars, the child's perception of horizontality was noted. Similarly, the child's efforts to draw houses, trees, and animals situated on the sides of mountains were used to determine his stage of development in terms of the vertical dimension.[5]

According to Piaget, Stage I is recognized by the inability to distinguish surfaces or planes, in the case of either fluids or solids. Substage IIA indicates that the water level is shown parallel with the base of the jar and the trees are perpendicular to the mountainside. Substage IIB is one of intermediate types of responses in which the child is aware that some system of reference is necessary to make his drawings "look right," but is yet unable to consistently and correctly find this reference. Stage III represents the child's successful mastery of both the horizontal and vertical aspects of his drawings.[6]

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[4]Piaget, op. cit., pp. 379-380.

[5]Ibid., pp. 381-382.

[6]Ibid., pp. 382-384.

Several studies have been devoted to those aspects of Piaget's theory which are under consideration here.

Carolyn Uhlinger Shantz and Charles D. Smock explain the theory thusly, "Piaget and Inhelder...postulated a sequential development of the child's representation of space, culminating in the acquisition of a Euclidean structuring of space. The co-ordinates of Euclidean space may be described as a grid of lines crossing each other perpendicularly in three dimensions. The presence of such a conceptual co-ordinate system is attested to when horizontality and verticality become independent of the perceptual properties of objects and immediate surround...It is Piaget and Inhelder's contention that the conservation of distance is a prerequisite for the development of the co-ordinate system."[7]

Shantz and Smock used a two-fold definition of distance conservation, stating there are "two cognitive operations involved...(a) distance must remain constant whether the space between two points is filled or empty, and (b) distance must remain identical between two points regardless of the direction of travel (AB = BA)."[8]

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[7]Carolyn Uhlinger Shantz, and Charles D. Smock, "Development of Distance Conservation and the Spatial Coordinate System," Logical Thinking in Children, Irving E. Sigel and Frank H. Hooper, eds. (New York: Holt, Rinehart and Winston, Inc., 1968), p. 158.

[8]Ibid.

These two investigators tested this theory using ten boys and ten girls from one first-grade class in a rural elementary school. The ages ranged from 6 years, 4 months to 7 years, 10 months. Five items using objects and five items using pictures were used to determine if each of the two requirements for distance conservation was met. To assess part (a), for example, "two 2-1/2 inch trees were placed before S 8 inches apart, and S was asked whether the trees were 'far apart' or 'near together.' A board higher than the trees was placed halfway between them and S was asked, 'And now, are they far apart (or near together)?' A child who conserves distance will assert that distance remains the same whether the space is filled or empty; non-conservers make a variety of responses, the most common being that the distance is less because the board 'uses up' some of the space between the trees."[9]

Among the tests used to assess part (b), the student was asked whether or not it is "just as far from here to here (left tree to right tree) as from here to here (right tree to left tree)."[10] The conserving child would recognize the equality of the distance, whereas the non-conserving child would assert the distance is greater in one direction.[11]

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[9]Ibid., p. 159

[10]Ibid., p. 160

[11]Ibid.

To determine if the child had developed use of a co-ordinate system, five tests were designed using first objects, then pictures, to assess each of the concepts of horizontality and verticality. Among the tests used for horizontality, the child was asked to indicate with his finger the location on an empty bottle of the water level should an identical partially-filled bottle be turned upside down. Among the tests used for verticality, a clay-covered stryrofoam pyramid, representing a mountain, with a 60^{o} slope was placed before the child. Figures with pins in their bases were to be placed "nice and straight" by the student at five sites verbally indicated. If the child had acquired the vertical concept, objects were placed on the mountain in reference to external axes, such as parallel to the walls of the room in which the tests were administered.[12]

The tasks using drawings, for example, yielded five students who used both concepts and seven who used neither. No child demonstrated command of the co-ordinate system who did not conserve distance. Eight students demonstrated distance conservation but did not successfully use the co-ordinate concept. Similar trends were noted from the tests

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[12]Ibid.

using objects. Thus, the data from the Shantz and Smock tests indicate a general consistency with Piaget's hypothesis of the developmental priority of distance conservation to the co-ordinate system.[13]

K. Lovell, D. Healey, and A. D. Rowland also tested children's ability to conserve distance in a study designed to replicate twelve of Piaget's experiments. The subjects used were as follows: (a) ten pupils in the 5-year-old age group, and fifteen in each of the age groups 6, 7, 8, and 9, totaling 70 primary pupils, and (b) ten pupils in each of the age groups 9, 10, 11, 13, and 14 in Educationally Subnormal Special Schools. All of these children were English in nationality.[14]

In this test "Two plastic 'Red Indians,' each 3 inches in height, were placed on a table about 20 inches apart. The child was asked if he thought the Indians were 'near together' or 'far apart.' When he had given his reply, the E placed a hardboard screen between the figures and asked the S whether they were still as 'near' or as 'far apart' depending on his first reply.

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[13]Ibid., pp. 161-163.

[14]K. Lovell, D. Healey, and A. D. Rowland, "Growth of Some Geometrical Concepts," <u>Logical Thinking in Children</u>, Irving E. Sigel and Frank H. Hooper, eds. (New York: Holt, Rinehart and Winston, Inc., 1968), pp. 141-142.

If the child was of the opinion that the screen altered the distance between the Indians, he was asked to give a reason. The screen was removed and replaced by a piece of wood 2 inches wide and 1 inch high (i. e., lower that the Red Indians). The same question was repeated in this new situation."[15]

To test the symmetrical character of the distance between the Red Indians, the child was asked, "Is it as far from there to there (AB), as it is from there to there (BA)?"[16]

Lovell, et al, found that the "main stages in the growth of certain geometrical concepts proposed by Piaget... have been broadly confirmed among English children... However, the number of children at the various stages were not always what one would expect from (Piaget's) book."[17]

P. C. Dodwell conducted a study designed to clarify a number of Piaget's studies pertaining to the child's understanding of spatial concepts. Of interest here is that portion pertaining to Piaget's theory that at stage A the child has no conception of horizontality and verticality; at stage B, the rudimentary concepts of horizontality and verticality appear but cannot be maintained with respect

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[15] Ibid., pp. 142-143.

[16] Ibid., p. 143.

[17] Ibid., p. 155.

to an external reference point, and at stage C, the concepts of horizontality and verticality have been mastered. Piaget places stage A at four or five years and "after seven to eight" for stage C.[18]

Dodwell describes his materials as "two narrow-necked bottles, one with straight parallel sides and a flat base, the other spherical, each about one-quarter full of a colored liquid; two other empty jars, one each of the types described; a cork with a matchstick fixed to it, and perpendicular to one of the flat faces; a plumbline with a bob shaped like a fish; a plasticine model of three mountains; and various pictures of the jars."[19]

Dodwell's procedure included such questions as follows: "We are going to tip the straight jar over like this... Will the water stay where it is, or will it move? Show with your finger on the jar where the water will be... Draw on your picture of the jar (tilted) where the water will be when it is tipped... Now we will see whether it is right... Were you right? Now draw what you see," and with the parallel-sided jar, "Here is a

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[18] P. C. Dodwell, "Children's Understanding of Spatial Concepts," Logical Thinking in Children, Irving E. Sigel and Frank H. Hooper, eds. (New York: Holt, Rinehart and Winston, Inc., 1968), p. 130.

[19] Ibid., p. 124.

cork with a matchstick in it. Let's pretend that it is a little boat. We will let the boat float on the water in the jar. Which way does the mast of the boat point now (Jar vertical.) We are going to tip the jar, and I want you to draw the way the mast will look with the jar tipped: I will tip the empty jar the way this jar is going to be tipped (45^{o}). Now draw the little boat... Now we will tip the jar with the water in it and see if you were right. Were you right? Draw it again the way it should look."[20]

Dodwell reports that all the substages and stages described by Piaget were observed, but his "findings suggest that the age limits cannot be so precisely identified, and the enormously preponderant 'mixed' category again suggests very little regularity in the developmental pattern."[21]

The current study is designed to further investigate the relationship between the child's ability to conserve distance and his skill in manipulating within a two-dimensional environment.

This writer offers no criticism of the tests used to measure conservation of distance. The test used in the current study is similar to that used by previous investigators although the models

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[20]Ibid., pp. 124-125.

[21]Ibid., pp. 130-131.

themselves differ. However, some discussion of the tests used to measure the co-ordinate system concept seems pertinent here.

In previously cited studies, the tests for the child's use of the co-ordinate system have been designed to measure first horizontality, then verticality. Piaget, Shantz and Smock, and Dodwell each used variations of water level tests for horizontality. Similarly, these investigators used variations of plumbline and mountain scene tests for verticality.

Piaget stated that the horizontality and verticality concepts are physical, rather than mathematical, and that the elaboration of these concepts poses a question independent of physics, that of the development of the co-ordinate system.[22] However, these tests are not independent of their physical orientation by their very nature.

Shantz and Smock also stated that the "co-ordinates of Euclidean space may be described as a grid of lines crossing each other perpendicularly in three dimensions. The presence of such a conceptual co-ordinate system is attested to when horizontality and verticality become independent of the perceptual properties of objects and immediate surround."[23] However,

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[22]Piaget, op. cit., p. 380.

[23]Shantz and Smock, op. cit., p. 158.

there seems to be a conflict between the definition of the co-ordinates of Euclidean space and the statement that it is the independence of horizontality and verticality from the physical surroundings which attests to the presence of the co-ordinate system. Not only are the tests used by Shantz and Smock not independent of the physical properties, but also they obviously fail to directly apply to the given definition. There is no occasion in which the child's ability to grasp the grid concept directly follows from the types of tests discussed above.

For the current study, it seemed more sound to design a model which would be in keeping with the truest mathematical interpretation of two-dimensions. That interpretation is perhaps best described as the Cartesian co-ordinate plane in which a grid of parallel lines crossing perpendicularly intersect in points with addresses in the form of ordered pairs of real numbers, (x, y). The x-co-ordinate indicates one dimension, and the y-co-ordinate indicates the second dimension.

This mathematical interpretation manifests itself in a model which is purely two-dimensional, regardless of the plane in which the model rests. That is, the child's concept of two-dimension can, in this investigator's opinion, be measured without requiring that his frame of reference actually be in terms of the horizontal and the vertical. Another distinct trait of this model is that it permits the measuring of both dimensions at one time, rather than requiring separate tests for each of the dimensions.

Thus, in the current study a test will be used which is unlike those employed by previous investigators. This test will utilize a model designed upon the mathematical interpretation of two-dimension discussed above. A thorough discussion of this test may be found in the next chapter.

II. HYPOTHESIS

Distance conservation does not precede two-dimensional awareness in the cognitive development of the child.

CHAPTER II

PROCEDURES

I. THE PILOT STUDY

A pilot study was conducted in March, 1971, in which the author individually interviewed thirty-seven children who attended Norman Smith Elementary School in Clarksville, Tennessee. Twenty-five of these children were seven years old; ten were eight years old; and two were nine years old. Of the total, twenty were girls, and seventeen were boys. These children were considered to be "average" by their teachers.

To test the child's grasp of two-dimensional relationships, two Cuisenaire Geoboards were placed before the child. The Geoboard is a commercially produced model used in the instruction of mathematics. The model is made of a hard plastic material molded into a square with a side of approximately seven inches. One surface has a twenty-five-peg lattice, as may be seen in Figure 1 of the Appendix. The investigator placed a metal ring around a spindle on one Geoboard, and the child was asked to place a similar ring on the corresponding spindle of the second Geoboard. This was performed a minimum of three times, with some children being asked to perform this task as many as six times.

Almost without exception the children successfully completed the two-dimensional tasks each time. Of one hundred thirty total opportunities, only fourteen errors were recorded. A possible reason for this is the small number of spindles from which to choose when using the Geoboard. As previously noted, the Geoboard has only twenty-five spindles, and most of the spindles chosen were not in the exterior rows or columns, leaving about nine choices.

To assess the child's ability to conserve distance, as defined by (a) in the study by Shantz and Smock, that is, distance must remain constant whether the space between two points is filled or empty, a square piece of plywood was placed before the child. The length of a side of the square was eighteen inches. At the approximate center of each of two opposite sides of the square was attached a thumbtack. The child was asked, "Are the two thumbtacks near together or far apart?" Using this answer as a reference, the experimenter placed a piece of plywood between the two thumbtacks in such a manner as to be perpendicular to the base, dividing the base into two approximately equal areas, and repeated the question.

The answer to the question as repeated frequently supplied no new information or insight into the child's thinking. For example, a child who had answered, "far apart" to the first question would likewise often answer "far apart" to the second question. The child perhaps recognized that the partitioning board actually took up very little of the total distance between the two thumbtacks. When the experimenter recognized this tendency, further questioning was used to determine if the distance between the thumbtacks remained the same, had become more or less, and why.

It was thought that perhaps the child would think the introduction of the partitioning plane would necessitate going around its left or right edge or over the top in order to go from one thumbtack to the other. This would, of course, probably make the child conclude that the partition caused the distance to be greater. Therefore, a second partition was then placed between the two thumbtacks where the first had been. This partition had a small door set on hinges in line with the thumbtacks. Once again the child was asked if the thumbtacks were "near together" or "far apart," when the door remained closed. The door was opened, and the child was further questioned about the distance between the thumbtacks.

There were occasions on which sufficient additional questioning was not used to clarify a child's answer, even though some responses were vague. However, seventeen children made definite statements regarding the greater or shorter distance between the two points after the introduction of one of the planes or the change in the position of the door. Only six children positively stated that there was no change in the distance.

The results of the pilot study indicated that more emphasis must be placed on determining exactly what a child means in order to correctly determine his ability to conserve distance. From the over-all responses, it appeared that the use of the partition with the door served to generate a number of inconsistent answers without identifying true conservation of distance. One child indicated that the mere presence of the door made the distance less even if it were not opened. To some children, opening the door decreased the distance, while others felt it increased the distance.

The preliminary investigation further illustrated the need to discard the Geoboard with its limited number of spindles in favor of a similar model with a greater number of spindles. This larger model was selected to help overcome the simplicity of the task when so greatly aided by the probability of chancing upon the correct response.

II. THE CURRENT INVESTIGATION

In April, 1971, the current study was made using a population of sixty second-grade students at Barksdale Elementary School in Clarksville, Tennessee. The ages ranged from seven years, five months to eight years, six months. There were sixteen seven-year-old boys, sixteen eight-year-old boys, eighteen seven-year-old girls, and ten eight-year-old girls. Barksdale does not homogeneously group its students; therefore, no particular characteristic of ability will be assumed. However, it should be pointed out that the area of Clarksville served by Barksdale School is one described as upper-middle class. According to the principal, there are eighty-eight families in which the head of the household is a professional, thirty-two of whom are teachers; forty families are supported by civil service employees; two hundred twenty-eight by business people; eighty by those in industrial management; twenty are self-employed, such as electricians and beauticians; the remaining twenty-eight families are supported by such people as retired military personnel and college students.

To determine the child's ability to manipulate in a two-dimensional situation, two models were especially designed. These models were identical squares of standard pegboard with ten rows and ten columns of holes which were one inch apart.

In each hole stood a wooden piece of dowelling approximately one and one-fourth inches high. The left margin of the square was outlined with a strip of red plastic tape; the lower margin of the square was outlined with a strip of blue plastic tape. (See Figure 2 in the Appendix.)

As in the pilot study, the investigator placed a metal ring around a spindle on one model, and the child was asked to place a similar ring on the corresponding spindle of the second model. This process was performed at least three times. Six children were given an additional opportunity, either because they merely wanted to do it again or because the investigator felt another occasion was warranted when the child intimated that he had made a mistake. Two correct responses out of three were again used as the criterion for being able to succeed in the two-dimensional task.

The base of the model used in the pilot study to test conservation of distance was also used in the actual experiment. It consisted of a square piece of plywood with a side of eighteen inches. In the approximate center of each of two opposite sides was a thumbtack. The child was asked if the thumbtacks were "near together or far apart." The experimenter then placed a piece of wood between the two thumbtacks in such a manner as to be perpendicular to the base, dividing the base into two approximately equal areas, as in the pilot study, and repeated the question. The partitioning board measured eighteen inches wide, twelve inches high, and three-fourths inch thick. (See Figure 3 in the Appendix.) This board was, therefore, about four times as thick as the partitioning board used in the pilot study. This change was made to reduce the chances of the child's saying the distance was the same, perhaps only because he thought the partition "took up" so very little space.

The child's answer to this second question pertaining to the thumbtacks being "near together or far apart," was then pursued to obtain a reason for the answer which he had given. If a child had answered "far apart" both times, he was questioned, "Is the amount of 'farness' the same, or is it greater, or is it less?" Regardless of which he answered, he was asked why.

This pursuit of the child's thought processes yielded nine children whose actual ability to conserve distance could not be adequately determined by this investigator. For example, one seven-year-old boy said after the introduction of the partitioning board, "both boards are two now, and they're closer." It is interpreted that the child referred to the two divisions of the base. He then indicated that the size of these divisions was the same, and the investigator failed to be able to communicate that these areas were not the real topic of consideration. Three other children had similar difficulties which could not be resolved even by placing the board at different positions between the thumbtacks.

A seven-year-old girl surprisingly recognized the relativity of the terms "near together" and "far apart." Therefore, it was not possible to establish the same type of frame of reference as in the rest of the population, and the investigator chooses to disregard this person in making the final analysis.

It is the opinion of the investigator that another type of problem resulted when there was recognized in certain children the desire to please rather than state their actual opinions. For example, if the child had perhaps answered "far apart" in the first instance and "near together" in the second, the answer was then changed to "far apart" again when asked to tell why. It seemed that the child was willing to say whatever he thought the investigator wanted to hear.

Of interest also are comments made by some of the remaining fifty-one children for whom complete data were secured. Among the non-conservers it was said, "the wood is wide and takes up room," "it blocks out some of the area," and "the wood is in the middle and splits it, so it isn't as much." Some of the rather outstanding remarks made by conservers to justify the distance as being the same before and after introducing the partitioning plane were "put the board in the middle, and the thumbtacks are in the same place," and "the wood (base) is still the same length and you just put it (partition) in line."

The reader is referred to the Appendix in which Tables III-VI show the age, sex, and type of response for each of the sixty children interviewed.

CHAPTER III

RESULTS

In considering the fifty-one children for whom the test data were complete, the following table indicates the results of the tests. The groups are arranged according to both age and sex.

TABLE I

RESULTS OF TWO-DIMENSIONAL SKILLS AND DISTANCE CONSERVATION TESTS GIVEN FIFTY-ONE SECOND GRADE STUDENTS ENROLLED IN BARKSDALE SCHOOL, CLARKSVILLE, TENNESSEE, 1970-1971, AFTER ELIMINATION OF NINE STUDENTS FOR WHOM DATA WERE INCOMPLETE

Age	Sex	Task 1	Task 2
7	M	12	2
7	F	11	3
8	M	13	4
8	F	7	4
Total		43	13

NOTE: This table should be read as follows: The number in the column labeled Task 1 indicates the number of students who were successful with the two-dimensional task. The number in the column labeled Task 2 indicates the number of children who conserved distance.

From this group of fifty-one children, twelve seven-year-old boys succeeded with the two-dimensional task while only two of them conserved distance, thirteen eight-year-old boys succeeded in the first task while only four conserved distance, eleven seven-year-old girls succeeded in the first task while only three conserved, and seven eight-year-old girls succeeded in the first task while only four conserved. This yields a total of forty-three children who possessed two-dimensional skills, and only thirteen children who identified distance conservation as here measured.

Of most interest here is an analysis of only those forty-three students who successfully performed the two-dimensional task. This analysis is to be made in terms of the child's ability or inability to conserve distance.

TABLE II

DISTRIBUTION OF FORTY-THREE CHILDREN ENROLLED IN BARKSDALE SCHOOL, CLARKSVILLE, TENNESSEE 1970-71, WHO SUCCESSFULLY PERFORMED TWO-DIMENSIONAL TEST IN TERMS OF ABILITY OR INABILITY TO CONSERVE DISTANCE

Sex	Age	Successful Two-Dimensional Skills	Conservers of Distance	Non-Conservers of Distance
M	7	12	2	10
F	7	11	2	9
M	8	13	4	9
F	8	7	3	4
Totals		43	11	32

It is to be noted that twenty-three seven-year-old children demonstrated skill with two-dimensional concepts, while only four, or 17 percent, conserved distance. Of these four, two were girls and two were boys. Twenty eight-year-old children demonstrated skill with two-dimensional concepts, while only seven, or 35 percent, conserved distance. Of these seven, three were girls and four were boys. Thus, of the total forty-three children under consideration, thirty-two were capable of manipulating within a two-dimensional environment but were unable to conserve distance.

Therefore, in view of the data reported here obtained in the manner described, this investigation fails to reject the hypothesis that distance conservation is not a developmental prerequisite for two-dimensional skills.

CHAPTER IV

CONCLUSIONS AND RECOMMENDATIONS

Table II on page 27 reveals a possible developmental pattern forming in the sense that the eight-year-old children demonstrated a higher percentage of conservers. Also, the differences between the sexes appear, to this observer, rather minor. For example, at age eight thirty-one percent of the males conserved, while forty-three percent of the females conserved. However, additional evidence would seem necessary to draw any valid conclusions about either of these possibilities, particularly considering the small number of children under consideration.

According to Piaget's developmental theory that distance conservation precedes two-dimensional skills, one would expect all forty-three children noted to have demonstrated the ability to manipulate in a two-dimensional environment to have also demonstrated the ability to conserve distance. Within this population, it is quite obvious that the number of children with this ability is much smaller.

There are at least three reasons for this failure to substantiate Piaget's theory. One reason may be inherent in the types of tests used. This investigator did not choose to pursue the child's understanding of the symmetrical aspect of the distance between two points, as defined by Shantz and Smock in (b), (that is, AB = BA). Nor did this investigator use as many different tests for distance conservation as has been done in previous studies.

The primary reason for the lack of agreement with Piaget's theory may be that this writer designed a model for testing two-dimensional skills quite unlike those models used by Piaget and others. It is to be admitted that the results of this test differ from those of Piaget possibly because of the nature of this model. However, it is the judgment of this investigator that true co-ordination of two dimensions can best be measured in the manner described for this study. Additional studies which would further test the appropriateness of this model are encouraged.

A second reason for this inconsistency with Piaget's theory is the possibility that this particular population has skills not necessarily present in the "average" child. Perhaps these children have been exposed to manipulative skills in such a manner as to unduly influence the test results. As noted earlier, these children tend to come from upper socio-economic homes. This implies their exposure to an intellectually stimulating environment. It is feasible that these children have enjoyed toys designed to increase their manipulative skills, while not stressing the conservation concept. Thus, through sheer practice, these children could score in the manner indicated. Investigations designed to analyze cultural influences seem in order.

The third reason is the possibility that Piaget could be in error when he says that distance conservation has developmental priority over use of the co-ordinate system. Certainly the need for more study is indicated by this possibility. Larger populations using a greater age range could be used to obtain information additional to that presented in this investigation.

According to Shantz and Smock, Piaget asserts that the "general age of emergence of distance conservation. . . (is) approximately seven years of age compared to emergence of the co-ordinate system at nine years of age."[24] The results of the current study indicate the emergence of distance conservation at a somewhat later age, noting that only eight of the total twenty-six eight-year-olds conserved distance. (See Tables IV and VI in the Appendix.)

The emergence of distance conservation has immediate consequences for the teacher of first- and second-grade students. The teacher is posed with the problem of determining the individual child's stage of development. If he fails to have reached the

[24] Ibid.

conservation stage, the teacher must decide if perhaps the child could attain this stage more rapidly than he naturally would if given some training. The possibility of training for conservation or any other developmental stage is itself a topic of much debate and is, therefore, worthy of considerable study. Whatever the teacher's conclusions concerning training, it would seem necessary to delay teaching or attempting to teach linear measurement, for example, until the child is a conserver of distance. It hardly seems feasible that a non-conserving child would gain any real understanding of measurement.

Another implication related to distance conservation is one which the teacher could readily execute. Once the teacher has chosen the time in which to teach measurement dependent upon the student's developmental stage, it seems of importance to this writer to include experiences with the symmetrical aspect of distance. The symmetrical aspect refers to the equality of the distance from point A to point B and the distance from point B to point A. Regardless of the unit of measurement which the teacher is using with the children, whether an arbitrary segment, a stick, the child's cubit measure, or the inch, the measuring experience can be had from each end of the distance to be measured.

The results of the current study seem to indicate that the teacher need not delay learning experiences with two-dimensional tasks until the child conserves distance. However, the point has been made that the interpretation of two-dimensional skills is made one way by this investigator and in a different way by Piaget. Therefore, it does not seem unreasonable to suggest that the teacher would be wise to include experiences of the type used by the current writer as well as those used by Piaget in his tests for horizontality and verticality. Because the elementary school child is usually led in an intuitive analysis of two-dimensions for the greater part of his early school years, practice with two-dimensional grids, water levels in bottles, plumblines, and the like, would tend to aid in his readiness for a more sophisticated study of the dimensional aspects of space at a later time.

CHAPTER V

SUMMARY

Piaget's theory that distance conservation precedes use of a co-ordinate system as a reference frame has been studied using a population of fifty-one second-grade children. Of the fifty-one students, forty-three were found to possess two-dimensional skills as measured by a model designed by the author. Only eleven of these were found to be conservers of distance.

Thus, using this investigator's interpretation of two-dimensional skills based upon the mathematician's use of the Cartesian co-ordinate plane and this investigator's test for distance conservation using a model of two points in a plane, Piaget's theory has not been substantiated for this population.

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APPENDIX

TABLE III

RESULTS OF TWO-DIMENSIONAL SKILLS AND DISTANCE CONSERVATION TESTS GIVEN SEVEN-YEAR-OLD BOYS ENROLLED IN BARKSDALE SCHOOL, CLARKSVILLE, TENNESSEE, 1970-1971

ID No.	Age Years	Age Months	Task 1	Task 2
1	7	7	x	
2	7	8	x	
3	7	9	x	*
4	7	10	x	x
5	7	10	x	
6	7	10	x	
7	7	10		
8	7	10	x	
9	7	10	x	*
10	7	10	x	x
11	7	10	x	*
12	7	10	x	
13	7	11	x	
14	7	11	x	
15	7	11	x	
16	7	11	x	
Total 16			15	2

NOTE: This table should be read as follows: An x appearing in the column labeled Task 1 indicates success with the two-dimensional task. An x appearing in the column labeled Task 2 indicates the ability to conserve distance. An asterisk (*) appearing in the column labeled Task 2 indicates the experimenter's failure to determine the child's ability.

TABLE IV

RESULTS OF TWO-DIMENSIONAL SKILLS AND DISTANCE CONSERVATION TESTS GIVEN EIGHT-YEAR-OLD BOYS ENROLLED IN BARKSDALE SCHOOL, CLARKSVILLE, TENNESSEE, 1970-1971

ID No.	Age Years	Age Months	Task 1	Task 2
17	8	0	x	
18	8	0	x	
19	8	0	x	*
20	8	0		
21	8	0	x	
22	8	0	x	
23	8	2	x	x
24	8	2	x	x
25	8	3	x	
26	8	3	x	
27	8	4	x	x
28	8	4	x	
29	8	5	x	
30	8	6	x	
31	8	6	x	x
32	8	6		
Total 16			14	4

NOTE: This table should be read as follows: An x appearing in the column labeled Task 1 indicates success with the two-dimensional task. An x appearing in the column labeled Task 2 indicates the ability to conserve distance. An asterisk (*) appearing in the column labeled Task 2 indicates the experimenter's failure to determine the child's ability.

TABLE V

RESULTS OF TWO-DIMENSIONAL SKILLS AND DISTANCE CONSERVATION TESTS GIVEN SEVEN-YEAR-OLD GIRLS ENROLLED IN BARKSDALE SCHOOL, CLARKSVILLE, TENNESSEE, 1970-1971

ID No.	Age Years	Age Months	Task 1	Task 2
33	7	5	x	
34	7	5	x	
35	7	5		*
36	7	6	x	
37	7	6	x	
38	7	7		
39	7	8	x	*
40	7	8	x	
41	7	8	x	
42	7	8		*
43	7	9		
44	7	9	x	
45	7	10	x	x
46	7	10	x	
47	7	10	x	
48	7	10		
49	7	11		x
50	7	11	x	x
Total	18		12	3

NOTE: This table should be read as follows: An x appearing in the column labeled Task 1 indicates success with the two-dimensional task. An x appearing in the column labeled Task 2 indicates the ability to conserve distance. An asterisk (*) appearing in the column labeled Task 2 indicates the experimenter's failure to determine the child's ability.

TABLE VI

RESULTS OF TWO-DIMENSIONAL SKILLS AND DISTANCE CONSERVATION TESTS GIVEN EIGHT-YEAR-OLD GIRLS ENROLLED IN BARKSDALE SCHOOL, CLARKSVILLE, TENNESSEE, 1970-1971

ID No.	Age Years	Age Months	Task 1	Task 2
51	8	1		*
52	8	2	x	
53	8	2	x	*
54	8	3	x	x
55	8	3	x	
56	8	3	x	x
57	8	3	x	x
58	8	3	x	
59	8	4	x	
60	8	5		x
Total 10			8	4

NOTE: This table should be read as follows: An x appearing in the column labeled Task 1 indicates success with the two-dimensional task. An x appearing in the column labeled Task 2 indicates the ability to conserve distance. An asterisk (*) appearing in the column labeled Task 2 indicates the experimenter's failure to determine the child's ability.

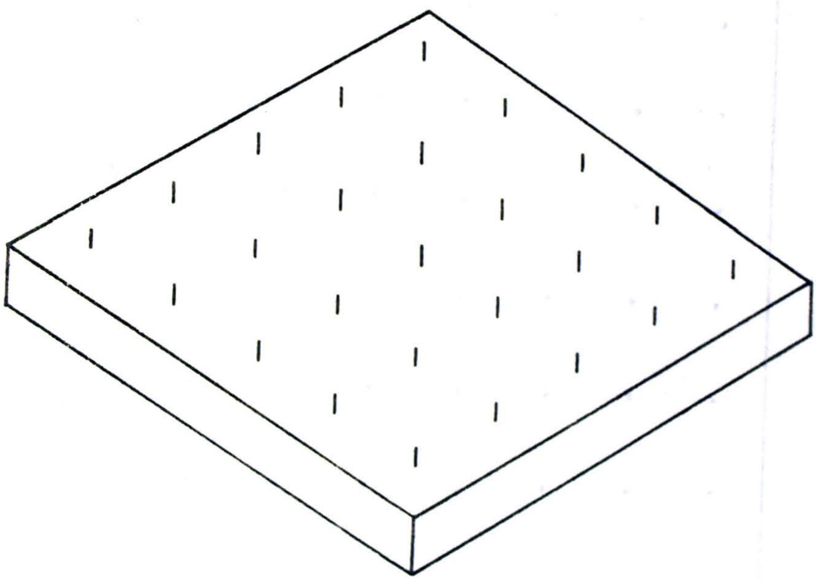

FIGURE 1

THE CUISENAIRE GEOBOARD

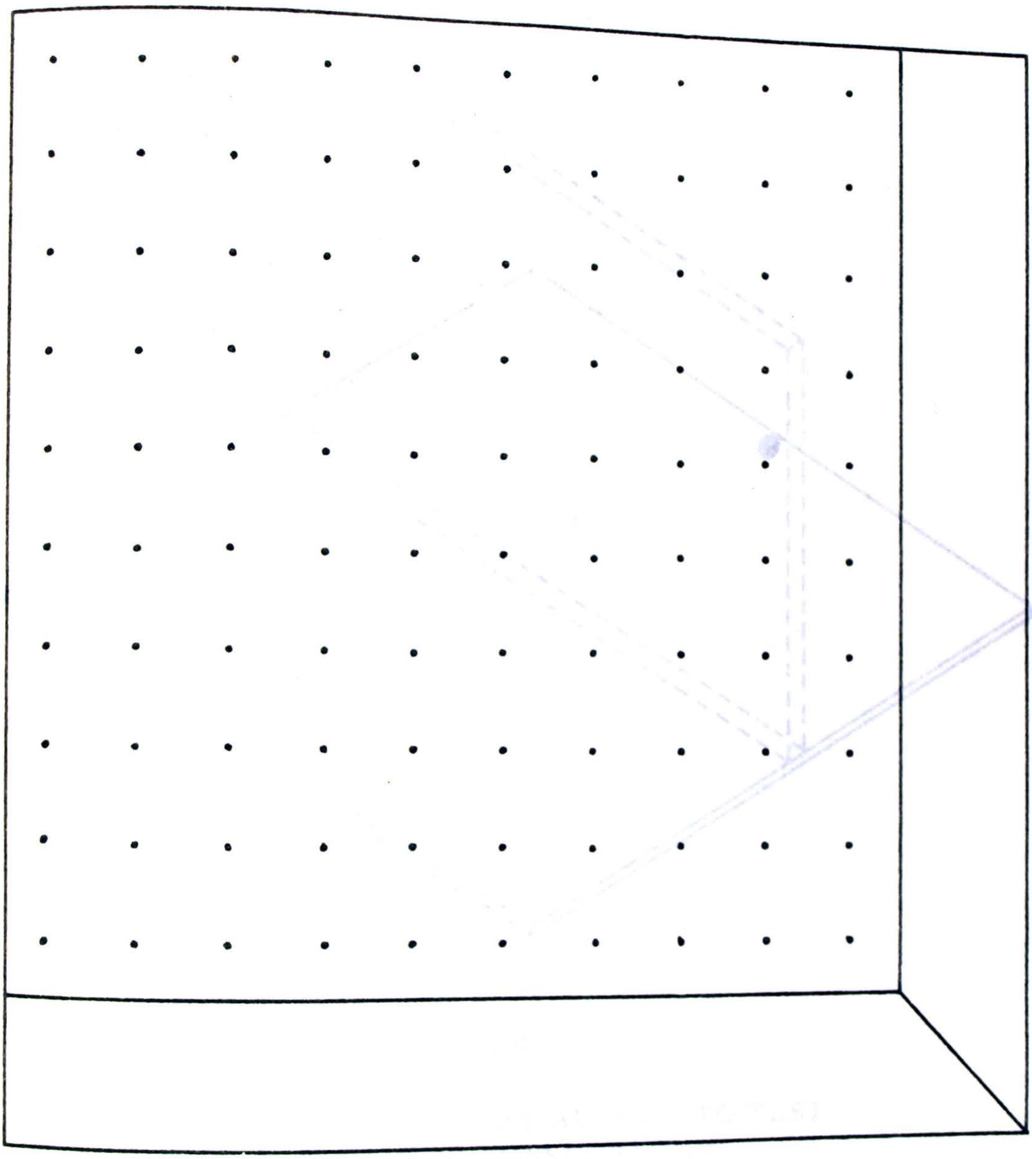

FIGURE 2

MODEL DESIGNED BY AUTHOR TO TEST TWO-DIMENSIONAL AWARENESS

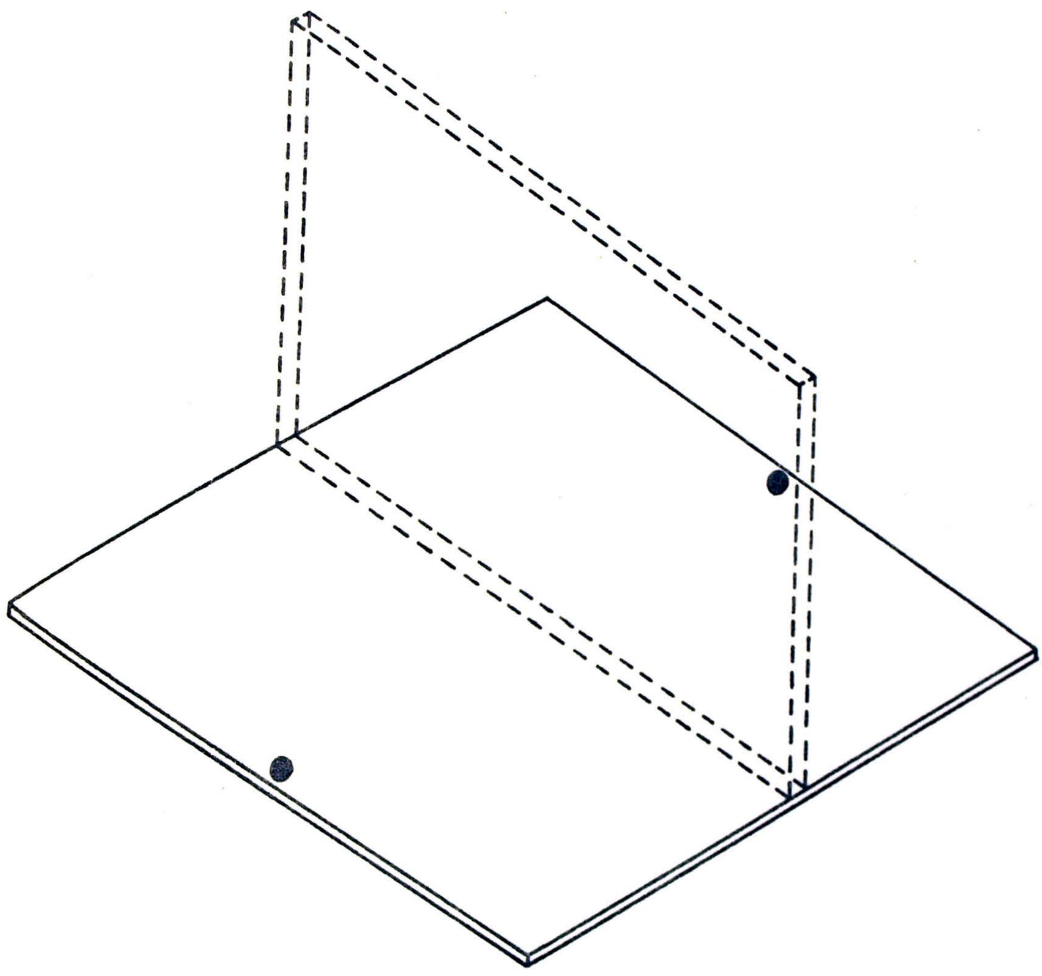

FIGURE 3

MODEL DESIGNED BY AUTHOR TO TEST
DISTANCE CONSERVATION