AN ORGANIZATION PLAN FOR THE TRANSITION FROM TRADITIONAL MATHEMATICS TO MODERN MATHEMATICS IN THE FOURTH GRADE

A Research Paper

Presented to

the Graduate Committee

Austin Peay State University

In Partial Fulfillment

of the Requirements for the Degree

Master of Arts in Education

by

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To the Graduate Council:

I am submitting herewith a Research Paper written by Mary Ann Downing entitled "An Organization Plan for the Transition from Traditional Mathematics to Modern Mathematics in the Fourth Grade." I recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Arts in Education, with a major in Curriculum and Instruction.

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Major Professor

Accepted for the Council:

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CHAPTER I

THE PROBLEM AND DEFINITIONS OF TERMS USED

The educational program of the school attempts to develop an inquiring mind in the student and to aid him in gaining the understandings, skills, attitudes, values, and appreciations related to becoming an educated person who thinks and acts effectively and is a contributing member of society. The goals of instruction in elementary school mathematics must be balanced with the total educational program of the elementary school.

I. THE PROBLEM

Statement of the problem. The purpose of this study is to design some procedures and techniques for introducing modern mathematical concepts to fourth graders who have been taught traditional mathematics. More specifically, answers were sought to the following questions: (1) What are the objectives of a fourth grade curriculum in modern mathematics?; (2) What mathematical concepts should the student entering fourth grade possess?; (3) How wide is the gap between the present program and the desired program?; and (4) How can the teacher implement the mathematics program to obtain the objectives of a modern mathematics curriculum?

Importance of the study. This fall, 1969, the teachers in Logan County, Kentucky will introduce what has been termed "modern mathematics" in their classrooms. A newly adopted textbook will be used at all grade levels. The school system has produced no guidelines or provided for any in-service training in procedures for introducing the new program.

The importance of the study derives from the instructional activities of the writer who is a teacher introducing modern mathematics to children who have been taught traditional mathematics. There is a need for a planned curriculum to introduce the basic concepts. It is hoped that this study will produce a tentative plan that the writer and other teachers in the school may use.

Limitations of the study. Information gathered for this paper was restricted to extensive reading of library materials in the field of new mathematics, to available research writings in the area, and to careful examination of several modern mathematics textbook series.

Assumptions:

1. There is a need to improve mathematics instruction in the elementary grades of Logan County.

2. Traditional mathematics teaching has failed to follow the meaning theory.

3. A deeper understanding of mathematical processes can be developed by learning the "why" of mathematics plus the "how."

II. DEFINITIONS OF TERMS USED

Algorism (or Algorithm). An algorism is both the pattern of numerals arranged for computation and the method by which the combinations are performed.

Array. An array is a specific arrangement of elements.

Associative Principle for Addition and Multiplication. This principle states that in adding or multiplying three or more objects, the addends or factors may be grouped in different ways without changing the result. $(6 + 7) + 5 = 6 + (7 + 5)$

Cardinal Number. A cardinal number tells how many objects a set has.

Commutative Principle for Addition and Multiplication. When adding or multiplying two numbers, the result remains the same when the numbers are interchanged. $2 \times 3 = 3 \times 2$

Distributive Principle of Multiplication with Respect to Addition. This principle asserts that the product of the multiplier and the sum of two or more addends is equal to the sum of the products of the multiplier. $6 \times 7 = 42$ or $6 \times (4 + 3) = (6 \times 4) + (6 \times 3) = 24 + 18 = 42$

Number. Numbers denote the concept of quantity which starts with the cardinality of a set of objects. Numbers cannot be seen or written.

Numerals. Numerals are symbols used to represent or name a number. Numerals can be seen and written.

Ordinal Number. An ordinal number is a number which indicates the position or order of a member of a set in relation to other members.

Set. A set is a collection or group of particular things.

Modern Mathematics. Mathematics of today is not as "new" as the name implies. What is "new" is the emphasis being given to topics not previously treated and especially the methods of teaching mathematics.

III. METHODS OF INVESTIGATION

Library research techniques represent the principal method of procedure used. An examination was made of numerous articles and books relating to the field of mathematics. Additional information was secured by examining mathematics textbooks both past and present. The writer also talked with several people in the field of mathematics about modern mathematics programs now in operation in other school systems.

IV. ORGANIZATION OF REMAINDER OF STUDY

After decisions were made as to what would be the extent of the study, the table of contents was developed as follows:

CHAPTER II

REVIEW OF THE LITERATURE

Numbers have been important in ages past, but our civilization is confronted with problems that involve an increasing knowledge of mathematics. Our very survival may be dependent on the solutions to these problems. Mathematics has been taught in American schools from the very beginning, but the reasons for teaching it and the goals to be considered have undergone many changes.

At the very beginning of the Colonial Period arithmetic was taught for utilitarian purposes. Its major use was as a tool in commerce, navigation, or particular trades.[1]

In the period 1787-1870 some consideration was still given to practical application, but the major objective during this period was mental discipline. During this same period there was an important change made in the method of achieving objectives. In 1821 Colburn published ARITHMETIC UPON THE INDUCTIVE METHOD OF INSTRUCTION, a book presenting Pestalozzi's ideas. Instead of memorizing rules as he had been doing, the student was now to set up his own rule as a result of his experiences.[2]

From 1860-1892 no essential changes were made in aim or content. It was late in the nineteenth century before real progress began to take

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[1]Eleanor Chastain, "Objectives of Experimental Courses in Elementary Mathematics," School, Science, and Math, Vol. 65, (January, 1965), pp. 49-55.

[2]<u>Ibid.</u>

place. In 1938 the Educational Policies Commission emphasized developing mathematics as enjoyment for the learner.[3]

Many people have the idea that the new mathematics phase began with the launching of Sputnik I in 1957. According to Eugene Nichols, the first impressive signs of serious dissatisfaction with the traditional mathematics curriculum preceded the launching of Sputnik by about six years. At the University of Illinois a group under the direction of Max Beberman concluded that the traditional subject matter of mathematics and the mode of its teaching at the secondary level needed a complete re-examination. The group began work on a new curriculum.[4]

Since that time numerous groups have contributed to the revision of the school mathematics curriculum. Educators and mathematicians became aware of the fact that revisions must also be made in the elementary school. Many curricula projects were begun. A brief description of the most important of these is as follows:

University of Illinois Committee on School Mathematics. Since its founding in 1951, UICSM has directed its efforts toward secondary school mathematics curriculum revision and the development of new teaching techniques. The project is under the direction of Max Beberman and has produced in loose-leaf notebooks a sequential curriculum for grades 9-12. The teaching innovations introduced by UICSM come under the heading

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[3] Ibid.

[4] Eugene D. Nichols, "The Many Forms of Revolution," National Association of Secondary School Principal's Bulletin, Vol. 52, (April, 1968), p. 17.

"discovery method." Since 1962 UICSM has moved into curriculum development for junior high students.[5]

Madison Project. The Madison Project was founded in 1957 at Syracuse University to promote more effective teaching of mathematics and to develop a supplemental program in math for grades K-12. An auxiliary center was started at Webster College, St. Louis, Missouri, and the project now has its main headquarters there. The director of this project is Robert B. Davis.[6]

School Mathematics Study Group. Founded in 1958 on guidelines established at a meeting sponsored by the National Science Foundation at the Massachusetts Institute of Technology in February, 1958, SMSG has had as its major thrust the preparation of sample textbooks to lead the way into the modern mathematics curriculum. Over sixty such texts have been written by SMSG teams or individuals. Current projects are aimed for K-12. The director of SMSG is E. G. Begle. The group is now located at Stanford University..[7]

Greater Cleveland Mathematics Program. The aim of GCMP has been to develop a comprehensive, sequential mathematics program for all those children in K-12, a program which is both mathematically sound and pedagogically correct. GCMP was begun in 1959 when the Advisory Committee of the Educational Research Council, a nonprofit organization whose purpose is to improve elementary and secondary education, asked the

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[5]*Ibid.*, p. 18.

[6]*Ibid.*, pp. 19-20.

[7]*Ibid.*, p. 23.

Council to direct its efforts toward improving the mathematics curriculum.[8]

There are several other projects under way for mathematics curriculum revision has become an ongoing process. Experimental groups are pouring out new ideas and materials with teachers and students testing them for teachability and providing the feedback for subsequent revisions. Running through all these projects are the following common threads.

1. Many obsolete topics deleted
2. New topics introduced
3. Teach more mathematics in less time
4. Utmost development of scientific potential of superior student
5. Increasing precision of mathematics language
6. Student participates more while learning mathematics
7. Student expected to develop ingenuity by discovering mathematical relations rather than be told
8. More emphasis on study and recognition of structural characteristics of mathematics
9. Direct involvement of mathematicians, mathematics educators, psychologists, researchers, teachers, supervisors, and administrators in planning curriculum reforms
10. Financial backing of government agencies, private foundations, local school systems, and local organizations is usual form of support for curricula reform efforts.[9]

Psychologists have recently pointed to studies that showed that the more the learner understood what he was doing, the faster he learned the material and the longer he was likely to retain it. This knowledge led to the developmental or discovery approach as the major method used for teaching mathematics today. Francis J. Mueller has this to say about the developmental approach.

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[8] Ibid., p. 26.

[9] Ibid., p. 35.

> The developmental approach leads the pupil through a series of graded problems and explanations that build up to some objective, say the subtraction of two large numbers, or the multiplication of two fractions, or perhaps an understanding of area. After this sort of development, the practice problems are introduced in smaller doses than previous texts. New educational theory holds that a few problems done carefully and thoughtfully have much greater instructional value than dozens of problems worked mechanically and with minimum thought.
>
> Today's pupils are encouraged to find alternative ways to compute and solve their problems. Teaching mathematics so that it makes sense to the learner has increased its attractiveness.[10]

Discovery is the key word in a modern mathematics program. The discovery plan is

> ...to have the student probe, speculate, and discover for himself generalizations that lie behind certain mathematical applications. Children are encouraged to search, to guess, and to ponder.[11]

The Madison Project materials are founded on the belief that good mathematics must be experienced, and the spirit is more important than the outward form. Heavy reliance is placed on group discussion by the children with the teacher serving as moderator or discussion leader.[12]

There are several teaching procedures that might make use of the discovery approach. Bert Kersh gives two of the most commonly used classroom procedures.

> One discovery approach results when the teacher guides the learner by revealing the mathematical principle through "hints" which are provided one at a time. Skillfully employed, the procedure effectively leads the learner into pathways of meaning and understanding where he otherwise might not venture or which he might overlook.[12]

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[10]Francis J. Mueller, Understanding the New Elementary School Mathematics (Belmont, California: Dickenson Publishing Company, 1965), p. 5.

[11]Ibid., p. 10.

[12]Robert B. Davis, Discovery in Mathematics (Reading, Massachusetts: Addison-Wesley Publishing Company, 1964), p. 8.

> The second method is opposite of the first. Although the teacher may not give the learner any hints which reveal the answer to him, the teacher is permitted to suggest alternative plans of approach in problem-solving strategies.[13]

It can be seen that much emphasis has been placed on teaching methods, particularly on the method of discovery where traditional mathematics emphasized drill. As one writer states, "If we have truly modern mathematics we must revise the way in which we approach the material which we teach."[14] There must be creative teaching in order to motivate students to be creative in the learning process.

Perhaps an old Chinese proverb best sums up the views of the proponents of the discovery method of teaching mathematics: "I hear, and I forget; I see, and I remember; I do, and I understand."[15]

In the traditional mathematics of the past the general objectives have been twofold:

> (1) To serve a functional need to prepare children for the life they are to live as adults and to enable them as children to use mathematics in the everyday world around them;
> (2) To develop, at least for some children, mathematical literacy. For the most part, this twofold objective was implemented by having children learn rules of computation. Later, efforts to bring meaning to these rules made it possible not just to have "little computers" but to have little computers whose computations made sense to them.[16]

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[13] Bert Y. Kersh, "Learning by Discovery: Instructional Strategies," Arithmetic Teacher, Vol. 12, (October, 1965), p. 415.

[14] Cecil B. Read, "New Wine in Old Bottles," School, Science, and Math, Vol. 61, (March, 1961), p. 163.

[15] "Third International Curriculum Conference," Arithmetic Teacher, Vol. 15, (May, 1968), p. 412.

[16] E. Glenadine Gibb, "Basic Objectives of the New Mathematics," Education Digest, Vol. 31, (December, 1965), p. 45.

New Math requires that concepts be formed before routine performance. The new courses stress self-motivation and promote the idea that the structure of mathematics is based on logic rather than social application. There is a continuous flow of ideas throughout the entire program. Children are motivated to look for patterns and relationships and to enjoy mathematics.

To implement these objectives of today, Gibb says teachers should: (1) Develop mathematical ideas. Basic mathematical ideas which have their beginnings in the elementary school include concepts of sets, number, operation, relation, function, proof, and some basic concepts of geometry; (2) Develop ability to solve problems; (3) Develop techniques of computation. When children use their understanding of numbers, operations, and the decimal system of numeration, they come to see different techniques for computing, not just "the way" imposed by their text or teacher; (4) Develop a child's creative ability. Instead of the teacher being a "fuel pipe" pouring in knowledge through drill, repetition, memorization, and rote learning, the teacher becomes a "spark plug" encouraging children to think for themselves.[17]

In reviewing the literature the writer found that there has truly been a revolution in elementary school mathematics. This revolution has caused a critical evaluation of present programs, a reshaping of goals, and an examination of our procedures in the light of what our students are doing, what they are capable of doing and what are desirable outcomes.

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[17] _Ibid._, pp. 46-47.

The goals and objectives of the mathematics program must be broadened. A new curriculum must be planned and new teaching methods employed. A curriculum plan will be discussed in Chapter III.

CHAPTER III

PROPOSED FOURTH GRADE MATHEMATICS CURRICULUM

In order to adequately provide the recommended program, the teacher of arithmetic needs to understand some of the basic principles of learning and how to apply them in selecting, organizing, and conducting learning experiences in arithmetic; in using effective means of meeting individual differences; and in evaluating pupil learning.

Following are some generally accepted principles of learning which apply to the teaching of arithmetic:

1. Readiness and motivation are important factors in effective learning.

2. Learning is understanding rather than mechanical memorization and involves seeing relationships and making appropriate generalizations.

3. Active participation and "discovery" through varied learning activities and with varied learning materials makes for more effective learning.

4. Learning is a developmental process by which the learner gradually reaches more mature levels of insight.

5. Individuals differ in their rate of learning.

6. Practice is necessary for proficiency and is more effective if preceded by understanding of the basic principle of what is being learned.

7. Retention, transfer, and application of learning are increased by emphasis on meaningful generalizations and on the application of generalizations in a variety of situations.

8. Knowledge of one's progress contributes to effective learning.[1]

Just as it is necessary for the teacher to know the basic principles of learning, it is also essential that he be familiar with some of the assumptions in curriculum planning. The programs of individual classrooms usually mirror the extent of the planning of these programs by teachers. It is the teacher's responsibility to make plans for the year, unit plans, and daily plans. The teacher must choose materials and select visual aids that will enhance the student's learning. As the teacher does this planning and selecting, he should keep in mind the following four basic assumptions in curriculum planning.

1. The curriculum itself must be dynamic and ever changing as new developments and needs in our society arise.

2. The process of curriculum planning must be continuous.

3. No master curriculum plan will serve all schools or all purposes.

4. Procedures of curriculum planning vary from classroom to classroom and from school to school, but they should be logical, consistent, and identifiable in each situation.[2]

OBJECTIVES OF A FOURTH GRADE CURRICULUM IN MODERN MATHEMATICS

There should be more to a mathematics program on any level than merely correct answers derived from memorized procedures. Mathematics

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[1]Frances Flournoy, Elementary School Mathematics (Washington, D.C.: Center for Applied Research in Education, Inc., 1964), p. 52.

[2]J. Galen Saylor and William M. Alexander, Curriculum Planning for Modern Schools (New York: Holt, Rinehart, and Winston, Inc., 1966), P.4.

is mainly concerned with ideas. It is more than addition, subtraction, multiplication, division, fractions, etc. Mathematics is a way of thinking, a way of arriving at decisions, a way of making predictions.[3] To help each child arrive at this concept of mathematics should be the major objective of the mathematics program on any level.

After careful examination of several of the major textbook series, the writer found there were a number of common goals in the modern mathematics program. A major goal of elementary school mathematics today is to assist students not only to know "how" to solve problems, but to have some understanding of "why" the processes used yield a correct solution. For example, suppose a fourth grade child is learning a technique for finding the product of two numbers such as 4 and 17. He may make 4 lines of 17 dots each, associating the pair of numbers with a physical model. Using his understanding of numeration he thinks of 17 as 10 + 7 and separates the arrangement of dots into 4 lines of 10 dots and 4 lines of 7 dots. Now he has two arrays and associates two products with them; namely, (4 x 10) and (4 x 7). Again using his understanding of numeration and basic facts, he expresses the product as the sum of 40 and 28. This sum 40 + 28, or 68, names the product of 4 and 17. The child is using the distributive property of multiplication over addition. As he develops skill he should advance to algorithms. In modern math if the child is unable to advance to the conventional algorithms, he still has a method that will yield the correct solution.

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[5]Daniel H. Sandel, "Teach So Your Goals Are Showing," Arithmetic Teacher, Vol. 15, (April, 1968), p. 320.

Contrast the above approach with an approach where a child is told first to write 17 and then to multiply 7 x 4, carry a 2 (tens), multiply 1 by 4 and add 2 to 4. In the new programs, the shortcuts are those the child is encouraged to make when he can--not those imposed on him by rules in a text or by his teacher.

Another major goal in updated programs is the discovery approach. All new series advocate this method. This is in essence, an inductive reasoning process whereby students are led through experiences; and they in turn are able to draw conclusions, generalize, and possibly make a feasible conjecture.[4] The discovery approach was discussed in more detail in Chapter II.

Challenging each individual in the classroom is a goal of all programs. In the past, common goals were held for all children at a particular grade level without regard to ability. Slow learners were expected to memorize, for it was thought that memorization was easier than understanding. Understanding should precede everything else.

Providing a differentiated curriculum for individual children carries with it the responsibility of making appropriate selection of materials and teaching methods suitable not only for different age levels but also for different maturity levels at the same age. This will be one of the teacher's most difficult tasks. To provide for individual differences and motivate each child is a real challenge for the teacher.

A fourth major goal of elementary school mathematics is to assist students to see the structure of mathematics. Mathematics is made up of systems with patterns and structure that tie the different facets to the system.

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[4] Ibid.

Another goal which demands attention is the use of set vocabulary. Sets give an element of sophistication to the elementary school mathe-- matics program. They provide a unifying thread which permeates mathematics from the kindergarten program through graduate school.[5]

There is the goal in an elementary mathematics program of developing control and proficiency of operational skills. Understanding of the concept should precede practice, but practice is still needed. Practice can be made enjoyable by the use of games and manipulative materials.

The writer feels a very important part of the school mathematics program is to create a climate in the classroom in which boys and girls will enjoy mathematics. This climate should provide students with frequent opportunities for success and create confidence within the students.

The goals given in the preceding paragraphs parallel very closely the stated main objectives as given in the fourth book of a leading textbook series. These objectives as stated include:

> (1) To develop in pupils understanding of and appreciation for the structure and patterns found in arithmetic and geometry; (2) To develop and maintain arithmetic skills; (3) To provide pupils with a thorough foundation in all aspects of pre-high school mathematics; and (4) To enable pupils to understand the why, as well as the how, of basic mathematics.[6]

Teachers must be aware of the objectives of a modern mathematics program. The stimulus-response, computational-centered arithmetic may have been adequate in the past but not in the space age of today. Technological advances have been so rapid that we can not even be sure

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[5] _Ibid._, p. 322.

[6] Eugene D. Nichols and others, _Elementary Mathematics--Patterns and Structures_ (New York: Holt, Rinehart, and Winston, Inc., 1966), p. vii.

that a particular arithmetic skill learned beautifully today will be of any consequence twenty years from now. For an example, think how often a bank teller of today adds a column of figures mentally. Most of the time he relies on the machine. The modern programs do not advocate forgetting about adding. The point they emphasize is that adding in a modern society is becoming less important. The concept remains extremely important; for a well-developed and understood concept can be permanently useful even in a world where skills are apt to change in importance from decade to decade.

In Logan County, the average student entering the fourth grade who has been taught mathematics three years from the traditional textbook should possess certain basic math skills and knowledge. Of course, variations would occur according to each teacher, her method of teaching, and each individual student. The beginning fourth grader should have knowledge of the following topics. These topics were taken from the third grade textbook that was used in classrooms this year and in recent years.

Number System

Meaning, reading, writing 1-, 2-, and 3-place numbers
Money numbers to $9.99; meaning of 0
Roman numerals through XII; Ordinals through "twentieth"

Addition of Whole Numbers

Meaning and maintenance (facts with sums to 18)
Ways to express addition; order of addends; adding 0
Addition of 2- and 3-place numbers without and with carrying
Column addition and checking
Relation of sum to addend; term "sum"

Subtraction of Whole Numbers

Meanings and maintenance (facts with minuends to 18)
Ways to express subtraction; subtracting 0; 0 as remainder
Subtraction when 0 is in ten's and one's place; term "remainder"
Subtraction of 2- and 3-place numbers without and with borrowing
Relation of remainder to minuend

Multiplication of Whole Numbers

Meaning of multiplication; relation with addition
Ways to express multiplication;; term "product"
Facts for 2's and 2; 3's and 3; 4's and 4; multiplication by 1 or 0
Pairs of multiplication facts; whole stories (multiplication and division)
Multiplication of 2- and 3-place by 1-place number with carrying
Money numbers

Division of Whole Numbers

Meanings of division; relation with subtraction
Ways to express division; term "quotient"
Facts with divisor or quotient 2, 3, 4, 1; check
Pairs of division facts; whole stories (multiplication and division)
Short division of 2- and 3-place by 1-place number; money numbers

Fractions and Mixed Numbers

Meaning, recognition, reproduction of one equal part of object, group or number
Definition of a fraction as a number
Fractions used with measures (1/2 inch and 1/4 inch only)
Meaning of number below fraction line
Relative sizes of fractions

Geometry

Recognizing and reproducing simple geometric forms: line, circle rectangle, square

Use of Literal Numbers

Use of letter n to stand for missing sum, addend, remainder, minuend, or subtrahend

Measures

Meaning of measures: linear, time, weight, dozen, calendar, liquid
Reference measures: finger inch, arm foot, arm yard or body yard

Problem Solving

Meanings of addition, subtraction, multiplication, division; likeness of groups
Use of dots, number line, and pictures
Differentiating processes
Telling why problems are a certain kind
Making problems: from pictures; from a story
Recognizing tricky words; telling story in own words
Testing answers; relation of answers to numbers given
Problems without numbers; using measures; oral problems[7]

The above listed topics are those to which most beginning fourth grade children in Logan County have been exposed. When this list is contrasted with the following one in which the writer has compiled only the new topics and concepts generally found in modern third grade mathematics textbooks, the reader becomes aware of the gap between the two. This differential is pronounced, particularly in relation to vocabulary.

Sets

Union of sets; subsets
Sets used to define multiplication
Set partitioning used to interpret division
Set notation (braces)

Numbers and Numerals

Meaning of number and numeral
Different numerals for a number
Reading and writing numerals through 9,999

Place Value

One-through-four-digit numerals
Expanded notation
Regrouping of numbers

Order and Relations

Cardinal and ordinal numbers
Equations and inequalities and their symbols

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[7]Guy T. Buswell, William A. Brownell, and Irene Sauble, Arithmetic We Need (New York: Ginn and Company, 1961).

Addition of Whole Numbers

Union of sets
Number line
Properties of addition
1. Commutative property
2. Associative property
3. Identity property
Addition without and with regrouping

Subtraction of Whole Numbers

Set separation
Inverse relation between addition and subtraction
Subtraction without and with regrouping

Multiplication of Whole Numbers

Defined in terms of sets
Using array patterns
Repeated addition interpretation
Skip counting on number line
Combinations through 9 x 9
Properties of multiplication
1. Commutative property
2. Associative property
3. Identity property
4. Distributive property
Multiplying by zero
Multiples of tens and hundreds
Two and three-digit multiplication without and with regrouping

Division of Whole Numbers

Finding a missing factor
Partitioning of sets
Arrays
Skip counting on the number line
Repeated subtraction interpretation
Combinations through 81 ÷ 9
Two and three-digit division with zero remainder
Inverse relation between multiplication and division
Distributative property

Fractional Numbers and Numerals

Equivalent subsets
Fraction numbers: halves through eighths
Fractional numerals
Informal approach to addition to fractional numbers

Measurement

Money
Time
Linear measure to half-inch
Weight
Temperature

Geometry

Point
Line segment
Ray
Angle
Triangle
Quadrilaterals

As we examine the traditional content for Grade Three and the modern content for Grade Three, these differences are the most outstanding: the addition of the topic of sets, the properties of the four operations, multiplication facts extended to 9 x 9, long division, extension of fraction concepts, more content in the areas of geometry and measurement, and the large change in vocabulary. No longer used are the terms "carrying" in addition and "borrowing" in subtraction. The word "regrouping" is now used for both processes. Other new terms such as commutative, associative, distributive, properties of numbers, and identity element are new, not only to the children, but also to many teachers. Many teachers must "unlearn" the methods and vocabulary previously used. Perhaps this explains why modern mathematics is received many times with more enthusiasm by the students than the teachers. Teachers must be willing to carefully examine their methods of teaching and honestly answer the question, "Am I providing an adequate mathematics program to meet the needs of the youth in our schools today, one that will also prepare him for his world of tomorrow?" In order to answer this question in the affirmative, this writer felt it was necessary to plan an outline of a program in modern mathematics that could be used to fill in the gap between where the student from a traditional school is, and where he should be, in light of the objectives and content of a modern mathematics program. This was the objective for the next section.

CONTENT OUTLINE AND METHOD OF PRESENTATION

The basic textbook to be used in the classroom was adopted by an appointed committee of teachers in Logan County. Other than the standard use of the adopted text, it was left mostly to each individual teacher to select the methods and procedures to be used in teaching mathematics in the classroom.

The organization of material in most modern mathematics textbook series is in the form of a spiral plan. A topic is begun in one grade and extended with the introduction of other and more difficult aspects of the topic in several later grades. Enough time should be devoted to any new topic in each grade so that understanding and competence result.

The problems of introducing modern mathematics for the first time in the fourth grade are increased because of this spiral plan of organization. Each grade builds on concepts established in previous grades. It is necessary, therefore, that the fourth grade teacher go back and teach mathematical concepts the student should have learned in the first three grades. The student must have this background and be familiar with the new vocabulary before he can proceed with fourth grade material.

When a new program is adopted, patience and consideration are needed from pupils, teachers, and parents. If parents are not properly prepared sometimes there is a hesitancy on their part to accept something new. Pupils are generally not ready for the major emphasis placed on the discovery method. Children who have known only traditional mathematics may be bewildered by the discovery method. They become so accustomed to following the teacher's lead, it is difficult for them to take the initiative. Close cooperation, planning, and study are required of all people involved in the new program.

The major portion of this planning and study falls upon the teacher. Some suggested steps that may be helpful for the teacher to follow in introducing a new topic in the mathematics class include:

1. Social Experience step presents a situation within the experiential background of children in order to introduce a process in a functional setting and to establish a purpose for the new learning.

2. Concrete Manipulative step utilizes materials of all kinds.

3. Visualization or Semi-Concrete step illustrates examples with pictures, lines, dots, circles, diagrams, charts, etc.

4. Abstract step is when problems are computed with number symbols alone.[8]

After examination of textbooks on the first three levels and careful examination of the adopted text on the fourth grade level, this writer selected the following content to be taught in the fourth grade

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[8]Wilbur H. Dutton and L. J. Adams, Arithmetic for Teachers (Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1961), p. 8.

classroom. Included in the outline of content are some of the basic concepts to be developed and also some of the teaching procedure to be used.

Sets

The basic ideas about sets provide a context within which many purposeful mathematical concepts can be unified. Understanding the set concept and set language is the first step in the meaningful development of mathematical ideas.

A. Definition

A set is a group or collection of things. The objects in a set are called the elements or members of the set. When sets are listed, the elements are enclosed within a set of braces. The set itself is indicated by a letter of the alphabet. It is not necessary for the various elements to be related in any way, and the order of listing is not important.

A set may have many elements, one element, or no elements. A set with no elements is called the empty set or the null set. The two notations for the empty set are braces with no element or $\emptyset$.

B. Cardinal Number of a Set

With each family of sets is associated a cardinal number, or a number that tells how many elements are in the set. The cardinal number of the empty set is 0.

C. Union and Intersection of Sets

The union of any two sets is a set which consists of all those members which are in one set, or the other set, or in both sets. Two sets such as A = [Mary, John] and B = [Ann, Donna, Rita] have no common members. If the two sets are joined, a new set is formed which contains the members of the two sets. The new set is called the union of sets A and B.

Two sets such as C = [1, 2, 3] and D = [2, 3, 4, 5], may have common members. Intersection of sets C and D is the set of all elements common to C and D or E = [2, 3].

D. Disjoint Sets and Subsets

When sets have no members in common they are disjoint sets. To find the sum of a pair of numbers two disjoint sets must be selected.

A set is a subset of another set if it contains no elements that are not in the other set. Every set is a subset of itself, and the empty set is a subset of every set.

E. Equivalent and Equal Sets

Sets are equivalent when a one-to-one correspondence exists between them, and all sets which are equivalent to each other form a family of sets. With each family of sets is associated a cardinal number.

Sets are equal when they contain exactly the same elements. Equivalent sets may or may not be equal.

In introducing the topic of sets, the teacher should begin with examples with which the child is familiar as a set of dishes or a set of books. A large supply of concrete material should be available for the children to work with in demonstrating sets. The flannel board and flannel objects are most useful in the teaching of sets. Since this is a new concept, the children should be provided with many opportunities to work with both physical and flannel objects.

Numbers and Numerals

A. Definition

A number is an idea of how many, a concept, an abstraction. It is what is thought in the mind.

A numeral is a symbol, a name for a number. The numeral "7" is a symbol for the cardinal number 7 of a set. Seven may be named in many ways: 7, seven, VII, 6 + 1, 9 - 2, 14÷2. All of these ways of naming the number seven are called numerals.

B. Ordinal Numbers

Ordinal numbers are those which tell order rather than how many. First, second, etc., are ordinal numbers.

Addition and Subtraction of Whole Numbers

A. Addition and subtraction with regrouping

The student entering the fourth grade should have learned to add and subtract. If the student can correctly reach the algorism, it is advised that the teacher only introduce the new terminology. The idea of regrouping should be stressed instead of using the terms "carrying" and "borrowing". If some students are having trouble with these processes, the step by step approach should be used to teach regrouping.

In an addition problem such as 19 + 4 = ?, 19 + 4 can be regrouped as (10 + 9) + 4 in the first step. Because of the associative property, (10 + 9) + 4 becomes 10 + (9 + 4). Next the ones are added, and 10 + (9 + 4) becomes 10 + 13. This step can be regarded as 10 + (10 + 3). Again, because of the associative property, 10 + (10 + 3) becomes (10 + 10) + 3 or (20 + 3). The child is given an opportunity to see that he will take the ten from the thirteen and group it with the other ten so that he has two tens and three ones, or twenty plus three. The last step is a final deduction from the previous steps: twenty is added to three to give twenty-three.

The inverse concept is important in developing the idea of regrouping in subtraction. Renaming is also used in subtraction. In doing a problem such as 25 + 8 = ?, the child learns to add thus:

$$\begin{array}{l} 25 = 20 + 5 \\ \underline{+\ 8 \qquad\ \ +\ 8} \\ \qquad 20 + 13 = 20 + (10 + 3) = \\ \qquad (20 + 10) + 3 = 33 \end{array}$$

To "undo" this problem, the child will learn to begin subtracting at the place where he finished adding. The equation 33 - 8 = ? can be written in expanded notation as:

$$\begin{array}{r} 30 + 3 = 20 + 13 \\ \underline{\quad -\ 8} \\ 20 + 5 = 25 \end{array}$$

When the child realizes it is impossible to subtract eight from three, he learns to think of another way to write

30 + 3. He must rename the 3 tens as 2 tens and 13 ones. When subtraction is taught as the inverse of addition, or "undoing" addition, the child becomes more aware of the relationship between these two processes.

When the child understands the concepts he will usually move to the traditional algorithm himself. He begins to think quickly in his mind without having to expand the problems on paper. This is the goal the child should reach. Some of the slow learners may not be able to advance to this step. They will need more review and more work with manipulative and semi-concrete materials until they understand and can utilize this concept.

B. Properties of Addition

The child from the traditional school will not be familiar with the names for the properties of addition even though he can work problems using these properties. Perhaps the best way to introduce these properties is again with the use of sets.

1. Commutative property--The child should be given experience with sets. He should realize that joining a set of three objects with a set of four objects gives the same result as joining the set of four objects to the set of three objects. Through many "doing experiences" the child is able to see the union of sets is a commutative operation. The children move from sets to the addition of numbers and through discovery they find addition is also commutative. Knowing this fact cuts in half the number of addition facts the child must know.

2. Associative property--This property is most often used in column addition. The children should work with sets first and then, as they are able, progress to the addition of three or more numbers. Fourth graders have usually checked column addition by adding up. The children should have ample opportunity to work problems as $(3 + 2) + 1$ or $3 + (2 + 1)$. They must learn the parentheses are used for grouping the two numerals to be added first.

3. Identity element for addition--The identity element for addition is zero. The students should know that zero added to any number always gives that same number.

Multiplication and Division of Whole Numbers

A. Meaning of multiplication and division

Children entering the fourth grade who have used only the conventional textbook will have had only the first four tables in multiplication, or they may not have reached multiplication at all. It will usually be necessary for the fourth grade teacher to start at the very beginning to present multiplication.

Multiplication may be thought of as repeated addition. The relationship of multiplication to addition is stressed by presenting the "doubles" in addition first: $2 + 2$, $3 + 3$, $4 + 4$, etc. The pupils quickly see that the addition sentence, $4 + 4 = 8$, can be stated as a multiplication sentence, $2 \times 4 = 8$. The number line should be used extensively to strengthen the meaning of multiplication in the mind of the child.

Division is relatively new to the fourth grader. The meaning of division may be thought of as repeated subtraction by relating it to the repeated addition. The number line once again proves useful as it graphically demonstrates another fundamental principle, namely, that division is the inverse of multiplication--that dividing undoes what is done by multiplying. The child discovers by making 3 moves of 2 spaces each that 3 x 2 = 6. He then undoes this operation by starting with 6 and moving back toward zero 2 spaces at a time. He finds he can make 3 such moves. At the same time, he discovers that division is related to successive subtraction.

B. Properties of multiplication

The commutative property of multiplication can be shown through the use of sets and arrays. Begin by reviewing the commutative property of addition. The child should see that since addition is commutative, and multiplication may be thought of as repeated addition, then multiplication is commutative.

Multiplication is a binary operation. When three factors are involved, the child must start with just two factors. The child discovers by experimentation that he can group the factors by twos in any way he chooses and always get the same product. The child should be led to discover for himself that multiplication is also associative. Should the child not reach this conclusion himself, the teacher, by careful guidance using physical and flannel-board objects, should direct the child's attention to this concept.

The identity element for multiplication is not the same as that for addition. It is necessary that the teacher point out that zero, the additive identity element, also has a special role in multiplication, because zero times any number is zero. The child's understanding of identity element should lead him to the conclusion that one is the identity element for multiplication because the product of any number and one is that same number.

C. The distributive property

The distributive principle is very useful in the teaching of multiplication. This principle asserts that the product of two numbers is equal to the sum of the products when the addends that make one number are multiplied separately by the other number. For example, perhaps the child knows the multiplication facts for the table of 4 but not for the table of 8. He can work the problem 9×8 by using the distributive principle as follows:

$$\begin{aligned} 9 \times 8 &= 9 \times (4 + 4) \\ &= (9 \times 4) + (9 \times 4) \\ &= 36 + 36 = 72 \end{aligned}$$

The child can work more difficult problems at an earlier rate than before by applying the distributive principle. The child who truly understands this principle should advance to the conventional algorism with no trouble.

D. Algorisms

An algorism is the way the problem is arranged for computation and the method by which the operation is performed.

As a review before the children move to algorisms, much work in multiplication and division should be done using sets and arrays.

To provide interest and enthusiasm, the children should make their own individual flannel boards and be provided with flannel objects to use in making sets and arrays. The flannel boards can be made quite simply by stapling a piece of flannel over heavy cardboard.

After mastering the multiplication facts using concrete and semi-concrete objects, the child is ready to extend his knowledge. For example, when pupils have discovered the distributive principle and used it in discovering basic facts, they then put this principle to work in multiplying tens and ones. They know through their understanding of numbers that 43 can be thought of as 40 + 3. Applying the distributive principle they find the product of 4 and 43 by multiplying 40 and 3 each by 4 and then adding the products: $4 \times (40 + 3) = (4 \times 40) + (4 \times 3) = 160 + 12 + 172.$

Much use is made of the distributive principle when fourth graders are introduced to multiplication by two-place numbers. If the child was working the problem 14 x 13, he might use the distributive principle first to solve the problem. The pupil should use the long form as an introduction to the short form, the conventional algorithm.

Division algorithms may be developed by first using sets, arrays, and the number line as was done in multiplication. The children need frequent experiences using manipulative materials.

By using the links between division and successive subtraction and division as the inverse of multiplication, the child may elect to solve the problem any number of ways. Alternative solutions should be encouraged by the teacher. For example, the problem $248 \div 4$ may be worked in the following ways.

$$\begin{array}{r|l} & 60 + 2 = 62 \\ \hline 4 & 240 + 8 \\ & \underline{240 + 8} \end{array} \qquad \begin{array}{r|l} & 50 + 10 + 2 = 62 \\ \hline 4 & 200 + 40 + 8 \\ & \underline{200 + 40 + 8} \end{array}$$

$$\begin{array}{rr|r} & 62 & \\ \hline 4 & 248 & \\ & 200 & 50 \\ \hline & 48 & \\ & 40 & 10 \\ \hline & 8 & \\ & 8 & \underline{2} \\ & & 62 \end{array} \qquad \begin{array}{rr|r} & 62 & \\ \hline 4 & 248 & \\ & 240 & 60 \\ \hline & 8 & \\ & 8 & \underline{2} \\ & & 62 \end{array}$$

E. Remainders in division

Remainders in division are introduced at the fourth grade level. Physical objects such as discs or blocks, arrays, and the number line prove to be helpful. The teacher should present problems with which the child is familiar such as, "How many nickels can you get for 47 pennies?" or "How many teams of 4 each can be formed if there are 17 children?" Seventeen of the pupils can form the teams and the child can see the number of teams formed and also how many children are left over.

F. Applications

The goal for teaching the four basic operations is that the pupil will be able to apply them in problem situations. The writer has found that reading problems present great difficulty to fourth graders because they do not know which operation to use to solve the problem. It is hoped, that in the modern program, the student not only learns the "how" but the "why." The student who truly understands the meaning of the fundamental operation will be able to correctly apply these learnings in solving problems.

Estimation

A. Rounding numbers

Rounding numbers to the nearest multiple of ten and hundred can be very useful when exact numbers are not needed. Numbers which are the result of counting are exact numbers; whereas, rounded numbers are only approximate.

A number line may be used to round a number. To round a number such as 34 to the nearest ten, the child can see on the number line that 34 is only 4 away from 30 and 6 away from 40. Thus, 34 rounded to the nearest ten is 30.

The teacher must present the idea that a number halfway between two multiples of ten is rounded up to the greater of the two. Thus, 35 rounded to the nearest ten is 40.

Rounding to the nearest hundred or thousand may be presented using a number line or number line segment. As the students gain in experience, they will no longer need the number line.

B. Estimating answers to problems

The children's interest in estimation may be stimulated by placing a jar of objects such as marbles, jacks, buttons or beans on a table and asking the students to estimate the number of objects.

The teacher can explain that estimated answers are easier to find and easier to remember than exact answers.

They may also serve as a check on an answer arrived at by computation.

Number Sentences

A. True and false statements

If a mathematical sentence or equation can be judged true or false, the sentence is called a statement; $3 + 4 = 7$ and $6 + 8 = 12$ are statements. Children must be taught to examine statements and judge whether they are true or false. Open sentences as $\square + 7 = 17$ cannot be judged true or false until the frame is replaced by a numeral.

B. Solution of equations and inequalities

All open sentences are equations that must be solved. The children should be given a large variety of equations that will aid them in their problem solving ability. The frame should be located in different places in the equation.

The fourth grader will be introduced to some new symbols concerning inequalities. The symbol $\neq$ means "is not equal to." The symbol $<$ means "is less than" and $>$ means "is greater than." Number sentences such as $10 < 11$ and $60 > 58$ are called inequalities. Most modern mathematics textbooks give a number of exercises involving inequalities.

Fractional Numbers

A. Meaning of fraction

1. A fraction is one or more equal parts of a unit. The child can make this discovery by cutting an apple or candy bar into parts.

2. A fraction may name one or more equal parts of a group of units. Four apples, candy bars, etc., can be displayed and the child removes 1 of 4. He removes 1/4 of the group.

3. A fraction is an indicated division. One-fourth is actually 1 ÷ 4.

B. Renaming fractional numbers

A fractional number line is almost a must in teaching equivalent fractions. The pupils should make their own number lines for easy reference.

The teacher must provide individual fraction kits or construction paper for cutting circles, squares, and rectangles into fractional parts. Flannel board fractional cut-outs should be available. All possible visual aids should be employed when working with fractions.

By using the fractional number lines and fraction cut-outs, the students can observe that 2/2, 3/3, 4/4,...are all names for the whole number 1. They discover the relation of fractional numbers to whole numbers. They can see there are many names for the whole number. Using the same procedures the child is led to the realization that 1/4 and 1/2 can also be renamed with equivalent fractions. The children should be able to observe patterns in making equivalent fractions.

The children are introduced to the terms simpler form and simplest form when referring to the equivalent fractional numerals. The common factor concept is presented and used in finding simplest form. To find the simplest form for 4/10 the child must find a common factor. This can be done by experimentation. The child knows he has reached simplest form when the only common factor is one.

C. Addition and subtraction of fractional numerals

In developing a concept such as addition of fractional numbers, the teacher should again begin with a physical model. The children can observe from a circle divided into sixths that 1/6 + 1/6 = 2/6. The same result can be accomplished by using pictures of number rays. After the children have observed several models and number rays, they should discover the pattern: $1/4 + 1/4 = \frac{1+1}{4}$, $1/6 + 1/6 = \frac{1+1}{6}$, $2/7 + 3/7 = \frac{2+3}{7}$. Once the child has this pattern in his mind he does not need to refer to physical models; he can supply the answer by applying the pattern of adding the numerators and using the same denominator.

The same procedure should be followed in subtraction of fractional numbers progressing from physical models to the abstract. The relationship between addition and subtraction should be emphasized.

D. Mixed numerals

A whole number and a fraction comprise a mixed numeral. By using number rays and fractional parts the child can discover the relationship between a mixed numeral and an improper fraction and how to change from one to the other.

Addition and subtraction of mixed numerals with common denominators is introduced. The concept of subtracting a fraction from a whole number is also included. One method for presenting this idea is to have the children make two circles. One circle is to be left whole and the other cut into fourths. The child can observe that 4/4 make the whole circle, thus 1 and 4/4 are different names for the same number. If the child takes away one of the fourths, he discovers that 3/4 names the part left. From this the child can move to the written problem:

$$\begin{array}{r} 1 = 4/4 \\ -\ 1/4 \\ \hline 3/4 \end{array}$$

This procedure should be repeated using squares, rectangles, etc., until the child has gained an understanding that can be applied without the use of visual aids.

Measurement

When teaching a unit on measurement it is essential that the class have access to the following materials of instruction: representative linear, liquid, and dry measures; scales, calendar, clock and thermometers.

A. Length

Linear measure may be introduced by having the pupils make rulers using non-standard units. They might do this by marking a non-standard unit segment such as a pencil repeatedly on a stick or pointer which they could then use to check the measurement of their desk. From an activity as this the child is able to see the need for standard units of measure. Several foot rulers and yardsticks should be available for the children's use and much actual measuring should take place.

B. Liquid measure

A cup, pint, quart, and gallon container are needed. The pupils can demonstrate with water or sand the relationships among the measures. At the same time they will gain both a visual and a kinesthetic impression of the size and the capacity of the commonly used measures. The pupils can make their own table of equivalent liquid measures.

C. Weight

Display several weighing scales of different types. A beam-balance scale would be helpful. Bathroom scales are readily available. Several children might weigh themselves. Others might find the weight of books and other objects. The beam-balance can be used to find the weight of a pencil in ounces and a pound. The children learn the ton is used to simplify the weighing of heavy objects.

D. Time

Using a display of several calendars, the pupils should observe and discuss (1) how a calendar is used in measuring time and (2) what the common units of time are. The pupils can construct a table showing the relation between days and weeks and between months and years. Making calendars would be a good activity.

The clock is another measuring instrument of time. The teacher should have a large model of a clock and individual model clocks can be made by the children. The teacher must lead the children to an understanding of the abbreviations m., a.m., and p.m. The children need practice in telling time to the minute and in measuring the amount of time that has passed or will pass between any two moments in time.

E. Temperature

Needed are several types of thermometers--vertical and circular thermometers used to measure temperature indoors and outdoors, cooking thermometers, and clinical thermometers. The children should demonstrate how heat and cold affect the reading on a thermometer. The children might keep a thermometer record of the temperature each day of the week.

F. Money

Money measurement may be introduced with a discussion of how change is made. Play money is desirable for the children's use. A store might be set up giving the students opportunity to make change and work money problems.

Geometry

A. Points and lines

The word, "point," in ordinary language suggests physical objects, such as the point of a pencil or the point of a pin. Using as many models as possible the child should be led to progressively smaller physical objects. From this they may begin to abstract the idea of a geometric point which is not a physical object. Pupils can be helped to understand that there are many models of a point--a dot on paper, the point of a pin, and so on. The dot on paper, like a numeral, merely represents an idea. The point itself, like a number, is an idea; and just as numbers are the building blocks of arithmetic, so points are the "building blocks" of geometry. Similarly, boys and girls must be helped to distinguish between a model of a line, drawn on paper, and a line itself, which is a set of points--and since a point is an idea, so is a line.

B. Closed figures--circles, squares, rectangles, and triangles

The pupils should be able to identify simple geometric shapes. They must also learn to recognize their characteristics, or properties. The pupil is allowed to discover many of these properties for himself as, for example, when he measures sides of a square and finds them equal in length; or when he measures line segments drawn from points on a circle to the center of the circle and finds them equal in length. Sources of enjoyment for the child lie in seeing geometric shapes in the things around him and in drawing geometric figures. Geometric concepts should be developed gradually, simply, correctly, and intuitively.

C. Finding the perimeter of quadilaterals

The teacher must first help the child understand the meaning of the word perimeter. To introduce finding the perimeter to children, tie together the ends of a piece of string (not more than three feet long). Have each of four pupils place a finger inside the loop and pull it taut so that the string represents the perimeter of a quadrilateral. Then cut the string and measure it against a yardstick to determine the linear measure of the distance around the quadrilateral. The pupils should suggest a shorter way of determining the distance around a closed figure.

PUPIL ACTIVITIES

In the average fourth grade classroom there is a wide range of ability. All the children will not be ready for the same concept at the same time. The teacher must determine the level of each child and then help him progress from that point. To aid in meeting individual differences of the students, the classroom could be divided into three or more groups according to ability.

The use of a single textbook cannot meet the needs of all learners. The teacher should have available textbooks on several grade levels. It is also important that the teacher supplement the textbook with varied visual materials and other printed materials. The teacher should provide worksheets and various teacher-prepared materials for use at all levels.

Mathematics should be a "doing" subject for all the students. A wide variety of commercially prepared and teacher-made materials should be available for use by all the students. The children can make their own flannel boards and counters. The more materials the children can make and the more they participate the more enthusiastic they are. Each child should have ample opportunity to work with manipulative devices and proceed to the abstract concept as his ability allows.

Practice to improve mathematical skill can be fun instead of boring with the use of games. The teacher may choose from a wide selection of mathematical games those best suited to improve the skills needed by each individual child or each group. Games can also be useful for motivation and enrichment purposes.

The teacher will find the overhead projector, films, and filmstrips useful aids in a mathematics program. The children will enjoy working their problems on the overhead projector. The teacher will find the projector quite helpful in demonstrating problems to a small group or the entire class.

Many times the classroom teacher will take the middle road and direct most of his attention to teaching the average child or the middle group. These children should definitely not be neglected and their needs should be met. It is equally important that the teacher provide instructional variations for the children at either extreme, the rapid learner and the slow learner.

Some of the instructional activities for the rapid learners should include: (a) more independent reading and use of the textbook; (b) using extra challenge worksheets; (c) using frequent and more varied mental arithmetic shortcuts; (d) participating in creative activities; and (e) reading supplementary books.

Instruction variations with slow learners include: (a) concrete experiences and materials to precede use of the textbook on a new topic; (b) very closely teacher-directed reading, study, and discussion of textbook material; (c) frequent dramatization of problem situations; (d) slower introduction of successive steps in a process; (e) frequent reteaching and review; and (f) more use of games to motivate learning.[9]

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[9]Frances Flournoy, Elementary School Mathematics (Washington, D. C.: Center for Applied Research in Education, Inc., 1964), p. 86.

The teacher should be able to select activities from both the lists above to meet the individual needs of the children who fall between these extremes. The pupils' activities in mathematics depend to a great extent on the materials provided with which they can work. The teacher has the responsibility to develop a learning laboratory for mathematics.

MATERIALS

A list of suggested materials are listed in Appendix A.

EVALUATION

Effective evaluation is an essential ingredient in planning new programs. Evaluative techniques should be used throughout the entire year to improve instruction. Evaluation, when properly used, gives direction and guidance to daily instructional planning and development.

Evaluation devices can determine the pupil's readiness for future instruction. The classroom teacher can diagnose individual weaknesses and strengths, and provisions can be made for individual differences by the proper use of evaluation

In order to have direction and coherence in his evaluation, the classroom teacher must have clearly in his mind, before starting to evaluate, the goals or objectives of his instruction. The following basic goals should be evaluated daily:

1. Does the student show growth and understanding with regard to basic mathematical concepts?
2. Does the student possess the necessary skills in performing mathematical operations and processes?
3. Is the student accurate and precise?

4. Does the pupil have an enthusiasm for and enjoyment of mathematics?
5. Does the child see the place of mathematics in our complex society?
6. Is the child able to use comfortably the unique language and symbolism of mathematics?[10]

The teacher should use a variety of evaluation instruments. The teacher frequently employs the teacher-made test as an evaluation device. The teacher should make sure this device tests more than just computational skills. The teacher-made test should also test meanings and understandings. It should test the pupil's ability in problem solving and his knowledge of mathematical symbols and vocabulary.

Teacher observation and personal interview is perhaps the most valuable evaluation technique. This is the only technique that enables the teacher to get at the thought processes being used by the pupil. Oral questions and class discussions are also valuable aids in evaluation.

Always at the teacher's disposal are the published evaluation materials such as the standardized achievement tests. The most recent editions of many of these tests have up-dated their mathematics sections to provide for the children in a modern mathematics curriculum. Published evaluation materials can be useful when properly used. Their importance should not be placed above the teacher's observations or well-prepared teacher-made tests.

In addition to evaluation of the mathematics program throughout the year, at the year's end the author plans to make a total evaluation

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[10] George F. Madaus, "Evaluation of a Mathematics Program," *Arithmetic Teacher*, Vol. 8, (December, 1961), p. 419.

of the program and in view of the findings, restructure the curriculum to correct the weaknesses and to increase the strengths. In making this evaluation, the author will use pupil's progress during the year as measured by teacher observation, teacher-made tests, and standardized achievement tests. This end-of-the-year evaluation should indicate the effectiveness of the instruction techniques and the choice of materials. With this knowledge an improved program of instruction can be planned for the succeeding year.

SUMMARY

In the preceding pages a brief outline of the content which the writer has selected to teach in the classroom this fall has been given. The units will be planned in more detail prior to their actual teaching. This proposed curriculum content was planned with the objectives of a modern mathematics program in mind. In the introduction of topics the children are encouraged to participate and discover mathematical concepts. Pupil activities are planned with individual differences in mind. One of the characteristics of a modern program is a more effective use of instructional aids. It is accepted that visualization is a primary aid to learning. The author has given a list of materials necessary in a modern mathematics program in Appendix A.

To be effective the program must be continuously evaluated. The teacher should use various evaluation instruments to judge the effectiveness of the materials and the method of teaching. The teacher's evaluation should result in improved instruction.

CHAPTER IV

SUMMARY

Realizing a need for improvement in mathematics instruction in Logan County schools, the school system adopted a modern mathematics textbook series to be used in the schools this fall. The writer, having taught fourth grade for several years using a conventional textbook, and having some knowledge of the content of a modern program, knew there was a chasm between the two. The writer felt that if teachers did not do an adequate job of building a bridge for the students to cross from traditional mathematics to modern mathematics, the students instead of gaining a deeper understanding and appreciation for mathematics would become hopelessly lost and confused.

This paper, by defining the objectives of a modern program and showing the gap that exists in content between the two, has then provided an outline of the material to be taught in the fourth grade to give the student the background he needs in the modern program. The paper has greatly aided the writer in planning a program to be used in the classroom.

A modern program reflects a new, deeper concern for how children learn. It helps pupils gain the skills, concepts, and language necessary for mastery of the more difficult topics in the secondary school. The children of today need a modern mathematics program if they are to live effectively in and contribute intelligently to the world of today. Today there are uses for mathematics that were unheard of or even thought about a few years ago. Those students of today who will help send man to planets will be those given a firm foundation in mathematical ideas and taught to yearn for more.

BIBLIOGRAPHY

BOOKS

Buswell, Guy T., William A. Brownell, and Irene Sauble. Arithmetic We Need 4. New York: Ginn and Company, 1961

Davis, David C. Patterns of Primary Education. New York: Harper and Row Publishing Company, 1963.

Davis, Robert B. Discovery in Mathematics. Reading, Massachusetts: Addison-Wesley Publishing Company, 1964.

Dutton, William, and L. J. Adams. Arithmetic for Teachers. Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1961.

DeVault, M. Vere, Roger Osborn, and Hazel B. Forester. Discovering Mathematics 4. Columbus, Ohio: Charles E. Merrill Books, Inc., 1966.

Eicholz, Robert E. and others. A First Look at Modern Mathematics. Palo Alto, California: Addison-Wesley Publishing Company, 1964.

________, and others. Elementary School Mathematics 4. Palo Alto, California: Addison-Wesley Publishing Company, 1964.

Flournoy, Frances. Elementary School Mathematics. Washington, D. C.: Center for Applied Research in Education, Inc., 1964.

Glennon, Vincent J. and Leroy G. Callahan. Elementary School Mathematics. Washington, D. C.: Association for Supervision and Curriculum Development, NEA, 1968.

Grossnickle, Foster E. and William Metzner. The Use of Visual Aids in the Teaching of Arithmetic. New York: Rambler Press, 1950.

Hartung, Maurice L. and others. Charting the Course for Arithmetic. Chicago: Scott, Foresman and Company, 1960.

McSwain, E. T. and others. Arithmetic 4. Atlanta: Laidlaw Brothers, 1965.

Mueller, Francis J. Understanding the New Elementary School Mathematics . Belmont, California: Dickenson Publishing Company, 1965.

Nichols, Eugene D. and others. Elementary Mathematics - Patterns and Structures 4. New York: Holt, Rinehart, and Winston, 1966.

Payne, Joseph N. and others. Elementary Mathematics 4. New York: Harcourt, Brace, and World, Inc., 1966.

Saylor, J. Galen and William M. Alexander. Curriculum Planning for the Modern School. New York: Holt, Rinehart, and Winston, 1966.

Suppes, Patrick. Sets and Numbers 4. Chicago: L. W. Singer Company, 1966.

PERIODICALS

Bacon, Harold M. "A View of the New Math," Education Digest, 31: 34-36, September, 1965.

Chastain, Eleanor. "Objectives of Experimental Courses in Elementary Mathematics," School, Science, and Mathematics, 65: 49-55, January, 1965.

Feinstein, Irwin K. "What to Expect When You Start a New Math Program," Nation's Schools, 76: 42-43, August, 1965.

Geisze, Wallace, Leroy Sachs, and Robert Wendt. "Modern Teaching Methods for Modern Mathematics," National Association of Secondary School Principal's Bulletin, 52: 129-140, April 1968.

Gibb, E. Glenadine. "Basic Objectives of the New Mathematics," Education Digest, 31: 45-47, December, 1965.

________. "The Years Ahead," Arithmetic Teacher, 15: 433-436, May 1968.

Hlavaty, Julius H. "A Message to Teachers of Elementary Mathematics," Arithmetic Teacher, 15: 397-99, May, 1968.

Kersh, Bert Y. "Learning by Discovery: Instructional Strategies," Arithmetic Teacher, 12: 414-417, October, 1965.

Lerch, Harold H. and Charles T. Mangrum II. "Instructional Aids Suggested by Textbook Series," Arithmetic Teacher, 12: 543-46, November, 1965.

Madaus, George F. "Evaluation of a Mathematics Program," Arithmetic Teacher, 8: 418-421, December, 1961.

Nichols, Eugene D. "The Many Forms of Revolution," National Association of Secondary School Principal's Bulletin, 52: 16-36, April, 1968.

Rappaport, David. "Historical Factors That Have Influenced the Mathematics Program for the Primary Grades," School, Science, and Mathematics, 65: 25-33, January, 1965.

Read, Cecil B. "New Wine in Old Bottles," School, Science, and Mathematics, 61: 163-174, March, 1961.

Rivera, Emilie. "Some New Approaches to Old Arithmetic," Arithmetic Teacher, 8: 291-94, October, 1961.

Sandel, Daniel H. "Teach So Your Goals Are Showing," Arithmetic Teacher, 15: 320-23, April, 1968.

"Third International Curriculum Conference," Arithmetic Teacher, 15: 409-412, May, 1968.

Ullsvik, Bjarne R. "Basic Learnings in Mathematics," Educational Leadership, 12: 199-204, January, 1955.

Weaver, J. Fred. "The School Mathematics Study Group Project on Elementary School Mathematics: A Progress Report," Arithmetic Teacher, 8: 436-439, December, 1961.

________ and E. Glenadine Gibb. "Mathematics in the Elementary School," Review of Education Research, 34: 273-83, June, 1964.

Willoughby, Stephen S. "What Is the New Mathematics," National Association of Secondary School Principal's Bulletin, 52: 4-15, April, 1968.

APPENDIX A

MATHEMATICS MATERIALS

Textbooks for fourth grade and other grade levels

Supplementary books

Flannel board and flannel board materials

- Numeral assortment
- Modern mathematics symbols
- Modern mathematics vocabulary
- Operations on sets and numbers
- Flannel board fractional parts

Hundred chart

Number line

Expanded notation cards

Peg boards and pegs

Place value charts

Counting objects (discs, popsicle sticks, bottle caps)

Fraction number line

Clocks

Scales

Rulers, yardsticks, tape measures

Play money

Thermometer

Abacus

Geometric shapes

Multiplication and division grids

Liquid measuring devices

Opaque and overhead projector materials

Filmstrips

Films