THE CONCEPT OF AREA IN MATHEMATICS: INCLUDED OR NEGLECTED?

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FRANCES SMITH BYRD

THE CONCEPT OF AREA IN MATHEMATICS: INCLUDED OR NEGLECTED?

An Abstract

Presented to

the Graduate Council of

Austin Peay State University

In Partial Fulfillment

of the Requirements for the Degree

Education Specialist for Elementary Teachers

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by

Frances Smith Byrd

July, 1980

ABSTRACT

The purpose of this study was to determine to what extent eighth grade students understood the concept of area in mathematics. A teacher-made test involving five important concepts of area was used to determine the extent of understanding of the concept of area in mathematics possessed by eighth grade students in Stewart County Schools.

A review of the literature revealed that within the last decade, educators have expressed much concern about students' lack of understanding and their misunderstanding of the concept of area. They stressed that most student misconceptions of area resulted from a lack concrete experiences with subconcepts of area, which helped develop understanding of the concepts of area.

Data for this study were collected through the cooperation of the Stewart County School System, Dover, Tennessee. The study was limited to those concepts measured by the instrument used.

Results from the study indicated that Stewart County eighth grade students ranked very low on their understanding of the concept of area in mathematics. It appeared they had been formerly taught area by a computational (formula) approach instead of a laboratory

approach. Therefore, it was the conclusion of this study that for the sample involved a laboratory approach (unit-counting, unit-filling approach) to the teaching of the concept of area would tend to make an important contribution to students' overall conception of area in mathematics.

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July, 1980

To the Graduate Council:

I am submitting herewith a Thesis written by Frances Smith Byrd entitled "The Concept of Area in Mathematics: Included or Neglected?" I recommend that it be accepted in partial fulfillment of the requirements for the degree of Education Specialist with a major in Elementary Education.

Bryan Crutcher, Ed

Major Professor

We have read this thesis and recommend its acceptance:

Second Committee Member

Ernest Woodward

Third Committee Member

Accepted for the Council:

William H. Ellis

Dean of the Graduate School

ACKNOWLEDGEMENTS

The author wishes to express her appreciation to Dr. Bryan Crutcher, major professor, Department of Education, for his counsel in the preparation for this study and his critical analysis of the manuscript; to Dr. J. Ronald Groseclose, minor professor, for his encouragement and interest; and to Dr. Ernest Woodward, committee member, for his influence in initiating the original interest in the study and for the many hours he spent in helping prepare the instrument (test) used in the study.

The author wishes to acknowledge the assistance of Mr. Van Riggins, Superintendent of Stewart County Schools, Mr. Bill C. Cherry, Supervisor of Instruction, the principals, and the eighth grade teachers of the three schools involved. Without their cooperation, this study would not have been possible.

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Chapter I

NATURE OF THE PROBLEM

An understanding of the concept of area is important for children if they are to succeed in mathematics.[1] Though it is a standard part of the curriculum from elementary school through high school, according to many educators, very few beginning junior high students have even an inkling of the concept of area.

Since 1970, articles began to appear in mathematical journals which expressed a concern over students' lack of understanding of how to compute and measure area (area measure). Basically, the articles all said the same thing--students do not understand the concept of area because they have not learned the subconcepts of area nor had sufficient concrete experiences when working with area in mathematics classes.

This researcher became interested in children's lack of understanding about area in the summer of 1978. The researcher, along with about twenty other teachers, had the opportunity to observe a two-week, special seventh grade mathematics class sponsored by the National Science

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[1]James J. Hirstein, Charles E. Lamb, and Alan Osborne, "Student Misconceptions about Area Measure," The Arithmetic Teacher, XXV (March, 1978), p. 16.

Foundation. The class was taught by Dr. Thomas R. Hamel, Associate Professor of Mathematics at Austin Peay State University. Dr. Hamel is a math educator whose primary interest is teacher education. During one particular lesson, students were given dot paper representations of 10 by 10 geoboards. They were asked to draw as many pictures as they could of noncongruent rectangles with perimeter of twenty. They completed the activity and concluded that the square shape gave the greatest area. Next, Dr. Hamel presented the following problem:

> Suppose that fifty feet of fence is available and I wish to use this fence to enclose a garden, rectangular in shape. How should I do this to have the largest possible garden?[2]

During the workshop, several different sized plots were suggested by the students. Among those suggested were a 5 by 20 plot and a 12.5 by 12.5 plot. Again, after calculating, a majority of the class decided that the square (the 12.5 by 12.5) gave the greatest area. But one unusually enthusiastic student Heidi, readily expressed her dissatisfaction with what had been decided. She could not understand why "her" 5 by 20 plot did not have the same garden space as the 12.5 by 12.5 plot. She stated in a very frustrated manner that they both used

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[2] Ernest Woodward, "Heidi's Misconception about Area and Perimeter," (Article to be published at a later date), Clarksville, Tennessee, 1979, p. 3.

fifty feet of fencing; therefore, the gardens would have the same amount of space. Also, that "her" 5 by 20 garden would just have longer rows, while the 12.5 by 12.5 garden would have shorter rows.

Heidi's previous mathematics teacher was one of the teachers observing the class with Dr. Hamel. She revealed that Heidi was a bright, enthusiastic student who had made A's in mathematics.

Dr. Ernest Woodward, Professor of Mathematics at Austin Peay State University, was also an observer of the special class. Dr. Woodward, a prominent elementary mathematics educator, wondered, if a bright student like Heidi could not functionally distinguish between area and perimeter, what about this concept with less academically talented and less motivated students?[3]

Having collaborated with Dr. Woodward on several occasions concerning this episode and with encouragement from Dr. Woodward, the researcher decided to delve deeper into the subject of area to see if any studies had been done concerning students' lack of understanding of the concept of area.

Upon reviewing the mathematical journals and methods books in mathematics, the researcher realized that educators were expressing much concern on the subject

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[3] Ibid., p. 1.

of area. Not only did the authors of these articles identify the problem, they suggested methods on how to properly teach the concept of area in mathematics.

STATEMENT OF THE PROBLEM

The purpose of this study was to research the statement that eighth grade students in Stewart County do not understand the concept of area in mathematics.

DEFINITION OF TERMS

For the purpose of this study, the following meanings were applied to these terms:

1. Area. A measure of the amount of surface contained in some plane region.[4]

2. Perimeter. The distance around a closed geometric figure, which is the sum of the length of its sides.[5]

PROCEDURES

The researcher did a review of the available literature concerning students' understanding of the concept of area in mathematics. Also, various methods of the

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[4]Kathryn Besic Strangman, "Grids, tiles, and area," The Arithmetic Teacher, XV (December, 1968), p. 668.

[5]Janet Jean Brougher, "Discovery Activities with Area and Perimeter," The Arithmetic Teacher, XX (May, 1973), p. 282.

teaching of area was included in the review. The materials examined were limited to those in Woodward Library, Austin Peay State University, Clarksville, Tennessee.

Prior to the study, permission was obtained from Mr. Van Riggins, Superintendent of Schools for Stewart County, to test all eighth grade students (Year 1979-1980) from the three elementary schools in Stewart County. The three schools involved were North Stewart Elementary, W. T. Thomas Elementary, and Dover Elementary.

With the help of Dr. Woodward, the researcher prepared and administered the test on the concept of area in mathematics (see Appendix B, page 51).

After evaluating the tests results, the author suggested methods for teaching area to students in grades 4 through 7. She gave these suggestions to the Supervisor of Instruction of Stewart County. The Supervisor will pass the suggestions along to the classroom teachers and use the information for planning in-service training for these teachers. Hopefully, the teachers will use the suggestions so that students will learn this important concept and develop this skill of area measurement.

SIGNIFICANCE TO EDUCATION

An article by Melfried Olson at the University of Wyoming stated in a few sentences what seemed to be the basic problem about students' lack of understanding of area and what to do to solve it. She stated that the concept of area did receive a time allocation within the mathematics continuum, but the method of teaching area was usually such that the students emerged from the instruction with the feeling that area was obtained by an algorithmic-computational approach. Alone, this approach did nothing to enhance the concept of area.[6]

Olson suggested that together with the algorithmic-computational ability to determine area, teachers should instruct students in the concept of area. Initially this could be done by the working with figures in a plane and suggesting the concept of area as "filling" of a figure. The students should be allowed to acually "fill" figures with square units and count them. These manipulative experiences not only helped students extend learning, but they built intuitive understanding of the concept of area.[7] This manipulative approach is illustrated by an ancient Chinese Proverb:

> I hear, and I forget;
> I see, and I remember;
> I do, and I understand.[8]

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[6]Melfried Olson, "Area," ed. Robert Todd and Thomas G. Teates, "Measurement Corner," <u>School Science and Mathematics</u>, LXXVIII (February, 1978), p. 163.

[7]<u>Ibid</u>.

[8]J. R. MacLean, "The Quest for an Improved Curriculum," <u>Readings in Geometry from the Arithmetic Teacher</u>, ed. Margeurite Brydegaard and James E. Innskeep, Jr. (Washington, D. C.: The Council of Teachers of Mathematics, 1970), p. 14.

Though it is presently the voiced opinion of mathematics educators that elementary and junior high students do not understand the concept of area, there seems to be very little concern among parents, classroom teachers or the general public as to whether students can work with "area." Since the general trend toward content in the mathematics leans toward the fundamental operations on sets of numbers, very little importance, if any, is placed on area.

Though proof of whether eighth grade students understand the concept of area is beyond the scope of this study, it is the author's opinion that sample testing of 129 eighth grade students from three different schools might present evidence which would encourage teachers within Stewart County in grades 4 - 6 to improve their methods for teaching area in mathematics. It was the objective of this study to determine to what extent eighth grade students in Stewart County understood the concept of area. The instrument (test) used to measure the extent is described later in the paper.

The study was limited to one grade, all beginning eighth grade students (Year 1979-1980) in Stewart County. A variable that was considered in this study was the students from the three different schools had different backgrounds in mathematics. No effort was made to control the variables since some of the students had previously gone to schools outside Stewart County and had probably had different textbooks from those of the Stewart County students. Therefore, the validity of the test was low and the generalizations limited to Stewart County.

ASSUMPTIONS

The assumptions based on this study were:

1. That the researcher was familiar enough with the concept of area and various methods of teaching area to evaluate the test data and draw valid conclusions.

2. All students had been taught area previously in grades 4 - 7. (Upon examination of the mathematics textbooks, in grades 4 - 7, the author found that area was a standard part of the mathematics curriculum in Stewart County.) The mathematic textbooks series used in Stewart County was Holt's School Program, Holt, Rinehart and Winston. This series was adopted in 1977 to be used as the basic text for all students in K - grade 8.

3. That the students had not had any formal teaching on the concept of area since the school year 1978 - 1979.

SUMMARY

Although there is much concern among mathematics educators concerning students' lack of understanding of the concept of area, there seems to be very little concern about it with the regular classroom teacher. This study was initiated as an attempt to measure to what extent 129 eighth grade students in Stewart County understood the concepts of area.

This study was limited in time, sample, and scope and was confined to the concepts contained in the test.

A review of the literature concerning area concepts, suggested methods for teaching area, and misconceptions students have concerning area is presented in Chapter Two. Chapter Three presents collection, presentation, and interpretation of data. The final chapter contains the summary, conclusions, and recommendations.

Chapter II

REVIEW OF THE RELATED LITERATURE

The author visited the Austin Peay library and reviewed thirty-four articles concerning area in mathematics. Most of the authors stressed the importance of teaching the concept of area.

Hirstein et al. emphasized that concepts and understanding of area were important for a child to acquire because it was the most commonly used domain of measure in every day life. Also, it was important to the child for a second reason vital to success in mathematics. Understandings about area were the base of many of the models used by teachers and textbooks to explain numbers and number operations.[9]

Hirstein et al. further stated that if the teacher was to depend on area understandings to provide an explanation or a rationale of operations with numbers, then it was important to know the extent of students' understanding of the concepts of area. Also, it was important to be able to identify some of the pitfalls and misconceptions that children exhibit while working with area.[10]

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[9] Hirstein, Lamb, and Osborne, op.cit., p. 10.

[10] Ibid.

According to Carpenter et al., the National Assessment of Educational Progress had completed its second assessment in elementary mathematics. The subjects were nine-year olds and thirteen-year olds. An overview of the results showed that only twenty-eight percent of the nine-year olds could find the area of a rectangle that was divided into square units, while seventy-two percent of the thirteen-year olds could calculate the area of the rectangle from the dimension of the sides. Only four percent of the thirteen-year olds could find the area of a right triangle, and only twelve percent could find the area of a square given one of its sides. It was the conclusion of this study that students appeared to be learning mathematical skills by rote manipulation and did not understand the underlying concepts.[11]

AREA: INSTRUCTION IN ELEMENTARY PROGRAMS

Moon, an educational specialist who was a former sixth-grade mathematics teacher, stated that the topic of area was included in most elementary programs. He also stated that the study of area often posed serious

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[11]Thomas P. Carpenter and others, "Results and Implications of the Second NAEP Mathematics Assessments: Elementary School," <u>The Arithmetic Teacher</u>, XXVII (April, 1980), pp. 46 - 47.

difficulties for learners. An analysis of the difficulties were as follows:

1. Students did not have an adequate understanding of what area was when the topics were formally introduced.

2. Many texts assumed that learners had an intuitive understanding of area. Concepts were usually introduced by pictorial examples with written explanations that had very little meaning for learners who had not acquired intuitive understanding.

3. Learners tended to confuse measures of length, area, and volume. Learners found it difficult to distinguish between these measures without the opportunity to have worked with concrete representation.[12]

Concepts and Subconcepts Neglected

Steinen, a university math educator at both secondary and elementary level, attacked the "modern mathematics" program. He noted that the content of many of today's programs was an improvement over that of the past, but in most cases the manner of presentation was not.[13]

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[12] Leland Moon, Jr., "Laboratory Experiences with Perimeter, Area, and Volume," The Arithmetic Teacher, XXII (April, 1975), p. 281.

[13] Ramon F. Steinen, "Useless Knowledge and How to Produce It," The Arithmetic Teacher, XVII (December, 1970), p. 671.

Steinen cited as an example an experience one of his young friends (a second grade student) had while enrolled in a program called "Modern Mathematics." She had shown him a worksheet (Figure 1, page 14). The child's response to the worksheet was just as he had predicted. Each figure was roughly the same size as that given. Each figure she had drawn was an attempt to duplicate those on the worksheet. According to Steinen, such an initial exposure to the shapes did very little to help a child learn the characteristics that were truly peculiar to each.[14]

Steinen also expressed concern for what would happen to the student in life outside the classroom. He saw no value in students knowing the formula for the area of a triangle, but who could not take a triangular-shaped object and determine what its base and height were. He suggested that rather than find the area of three different triangles, a student would be far better off to find the area of a triangle three times, each time using a different side as its base. He maintained that when this method was used, the child worked with a problem with a built-in check. Also, the child would gain some insight into the nature of measurement and its limitations.[15]

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[14] _Ibid._

[15] _Ibid._

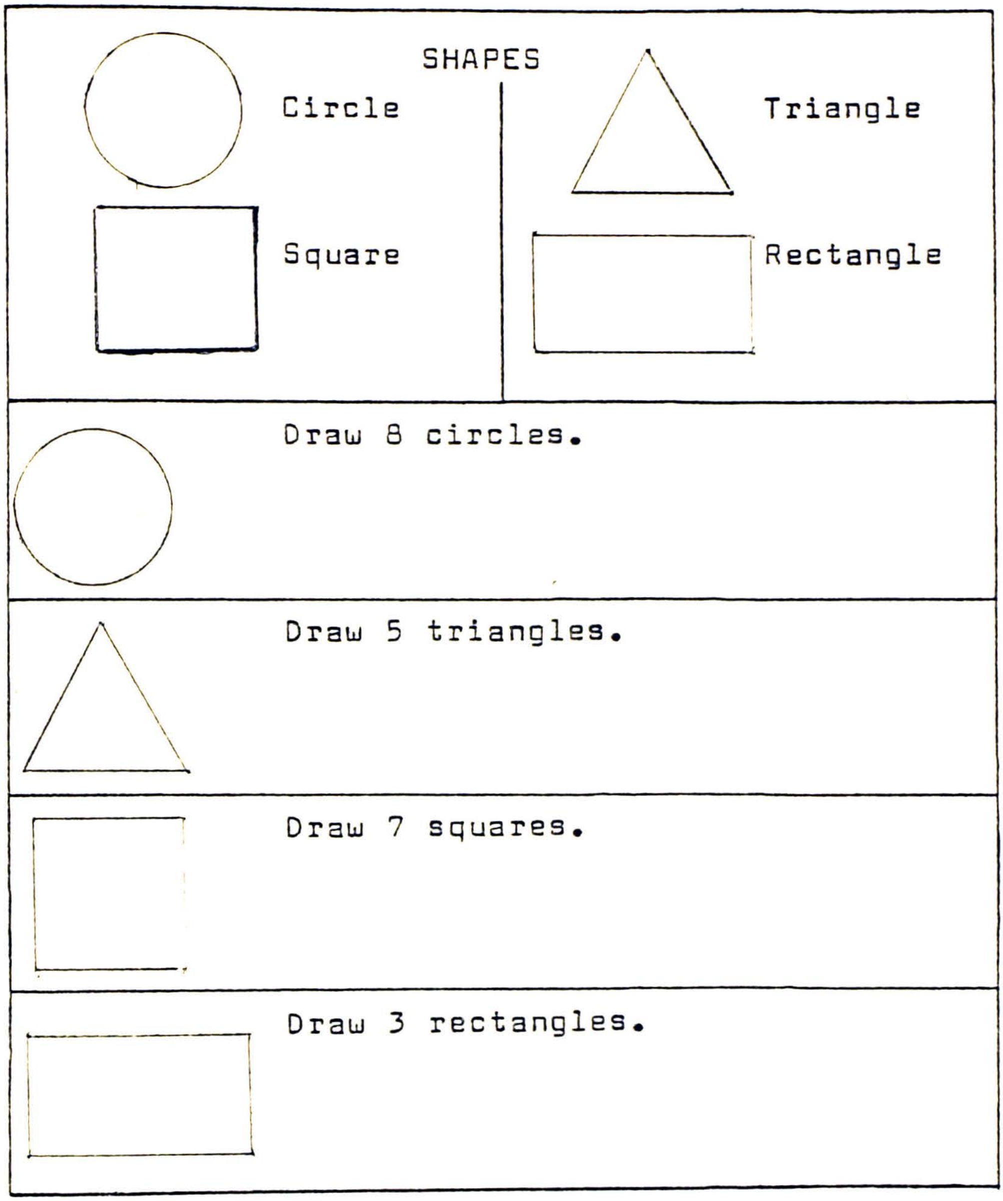

Figure 1

SAMPLE WORKSHEET

Steinen advocated it was not so much to "teach" the child as it was to provide opportunities for him to learn.[16]

Nelson and Reys revealed that area was often slighted by a too rapid progress to formula-based computation. Also, the child needed considerable experience with the primitive subconcepts in order to develop an intuitive feel for area. Only after the child developed an intuition for area, the problem for establishing the formulas for squares, rectangles, triangles, trapezoids, and circles are treated more readily.[17]

Jamski recently emphasized that superficial manipulation of formulas should not be equated with an understanding of the concept of area. Too often junior high school students were evaluated on their mastery of the concept of area with only superficial recall testing. Even at this level, the students should study the concept of area in a more challenging manner by exploring given figures by means of subdivision into non-overlapping parts that may or may not be squares.[18]

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[16] Ibid., p. 672.

[17] Doyal Nelson and Robert E. Reys, "Measurement in School Mathematics," 1976 Yearbook of the National Council of Teachers of Math, (Virginia), p. 24.

[18] William D. Jamski, "So Your Students Know About Area?" The Arithmetic Teacher, XXVI (December, 1978), p. 37.

Nelson et al. pointed out the idea of a unit for the area of a region began with subdivision. The child must be able to conceive a region as a union of subdivisions connected at the boundaries. The difficulties with this conception resulted from two requirements. First, the child must have accepted that the whole region was composed of subregions and also realized that the component parts could be taken apart and then reassembled to give the original shape. The second requirement was that the component parts must have had the same area. Other educators have studied the development of this concept. Their conclusion was that the parts-whole aspect took longer to develop and once it was developed, the matter of equal-sized parts developed relatively easily.[19]

Nelson et al. further stated that once the concept of a unit was present, the next step was the iteration of the units to assign a number to a given region. Most studies have employed regions that were to be covered with tiles, either square or triangular halves of squares. Children's ability to iterate area units have been studied by two methods. In one, the child was given enough identical tiles to cover both regions to be compared. In the other, only a few tiles were available. Therefore, the subdivision could not be made by completely covering the region. However, the child was given a

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[19]Nelson and Reys, *op.cit.*, p. 50.

pencil and was allowed to mark the regions. The conclusions were that (1) a sequence of abilities occurred that paralleled the development of length measurement, and (2) competence was attained for both area and length at about the same age.[20]

According to Nelson et al., other educators have disagreed with the second of these findings about the simultaneous acquisition of the notion of a unit of length and a unit area. They employed algorithmic procedures and tried to teach six and eight-year-old children to use unit iteration in both length and area situations. The older children were successful, but the younger children could learn only the length procedure. The difficulty seemed to be the inability of the children to consider two directions at the same time, which was necessary in the iteration on area units.[21]

Nelson et al. recalled that some psychologists found that children did not think of the area of a rectangle in terms of the product of length times width until about twelve or thirteen years of age.[22]

Unit counting was not without its drawbacks. Although it was suited for children's preliminary encounters with area, Nelson and Reys claimed that it

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[20] Ibid.

[21] Ibid.

[22] Ibid., p. 51.

sometimes led to difficulty. The fundamental idea of covering was natural and simple. It allowed a child to use counting skills in acquiring intuition about area. But it was when the child reached the stage of needing to deal with incommensurability that the counting or tiling basis of area presented difficulty. Instructional sequence was typically designed to evade this difficulty because resolving it took considerable mathematical maturity.[23]

CONCRETE EXPERIENCES WITH UNITS

In his textbook on teaching mathematics, Starr explained that area was one of the hardest mathematical concepts for children to understand and a great deal of time should be spent in building the concept. Students needed to understand that area was the surface coverage of an object and the units of measure used were in terms of square inch, square foot, and square miles.[24]

Starr emphasized that the understanding of these measures really came from actual manipulation of these units. When students actually measured surfaces by using units, they developed the concept of area and came to

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[23] Ibid., p. 26.

[24] John W. Starr, The Teaching of Mathematics in Elementary School (Pennsylvania: International Textbook Co., 1969), pp. 351 - 352.

realize why areas are labeled as square units rather than plain inches, feet, or yards.[25]

Rosenberg, a classroom teacher, claimed that students needed an introduction to the topic of square measure before formally working with area. He reported the following concrete experiences (activities) he had successfully completed with his seventh grade students:

1. Students made one-inch squares. Then two-inch squares were made. Through manipulation the students realized that there were four square inches in a two-inch square. Further manipulation led to the discovery of a formula for the area of a square.

2. Each student was given graph paper on which to trace one of his hands. He found the square measure of his hand by counting the total number of squares that were within the outline of the hand.

3. For a follow-up activity, each student had to trace and measure the area, in square inches, of one of his feet. This was done on blank paper. The students traced squares on the figures to be measured until the whole figure was filled. Then the students totaled the area of the included squares.

4. One student's body was outlined on a piece of large graph paper. For homework, the other students were

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[25] Ibid., p. 352.

assigned the problem (on blank paper) of finding "how square" their friend was. The same method was used as in the problem of measuring the foot except larger squares were needed. (The side and back of the body were not measured. The purpose was to develop skill in using square measure on a flat surface.)[26]

Filling and Covering with Units

Olson maintained, also, that mathematical computation of area could be expected of elementary children. The method of finding areas by "filling and covering" with unit squares was appropriate to enhance the concept of area. This method, when used with rectangles, provided exercises in multiplication and counting and served as a readiness activity prior to the development of a formula for finding the area of a rectangle. The use of geoboards and graph paper were useful in extending this area activity to triangles and other polygons that could be made using a geoboard. However, for some figures, the "filling, covering, and counting" method yielded errors no matter what square unit was chosen.[27]

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[26]Howard Rosenberg, "What's the Area?" ed. Charlotte W. Junge, "Things You Can Try," The Arithmetic Teacher, XVIII (October, 1971), pp. 429 - 430.

[27]Olson, op.cit., p. 166.

Area Concept and Manipulation

According to Szetela, other educators concluded that if shortcuts for finding area are introduced too early, the concepts will not be understood. Also, they observed that authors of recent elementary texts did recognize the need for concrete measuring experiences, but that problems on a printed page did not provide sufficient experience with area. Therefore, they suggested hands-on experiences with physical models for partitioning regions into squares. The teacher then needed to follow up the activities with problem-solving situations that were not only interesting, but also lead students to focus on the attributes of area.[28]

Based on experienced teaching seventh and eighth grade students, Szetela suggested that the concept of area needed to be developed with more active learning experiences, some of which included interesting problem solving situations.[29]

Moon contended that to alleviate difficulties encountered by students studying area, laboratory activities needed to be designed to introduce the topic. Each activity used should feature manipulation of concrete materials designed to build an intuitive understanding of

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[28]Walter Szetela, "Analogy and Problem Solving," The Arithmetic Teacher, XXVII (March, 1980), p. 19.

[29]Ibid., pp. 19, 22.

the topic of area. Each experience planned should provide the learner sufficient experience with concrete representations of standard measures of area and length to enable him to distinguish between the two.[30]

The concepts of area should be informally introduced as counting activities. Paige et al. recommended that tangrams be used to explore the concept that the shape of a figure can change without changing the area. (In tangrams, the same pieces were used to form new shapes.)[31]

Grids and Area

Strangman reported an approach to teaching area that was analogous to the way in which linear measurement was carried out. Therefore, it was a method that followed easily from the child's previous experiences.[32]

In this approach area was defined as a number; namely, as a number assigned to a region. (It was usually defined as a measure of the amount of surface contained in some plane region.)[33]

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[30]Moon, _op_._cit_., p. 281.

[31]Donald Paige and others, _Elementary Mathematical Methods_ (New York: Wiley, 1978), p. 20.

[32]Strangman, _op_._cit_., p. 668.

[33]_Ibid_.

The first step that Strangman presented was to choose a measuring unit. Although other units may have been used, such as a triangle, rectangle, hexagon, or other region, it was convenient to choose some square region which represented one square-inch.[34]

The next step was one of comparison. The plane region to be measured was completely covered with the square units. Next, copies of the measuring unit were placed side by side, and row upon row, thus, a grid was obtained. The grid became the measuring instrument. The measurement problem then became one that determined how many of these measuring units were needed to completely cover the given region.[35]

The grid was then duplicated on translucent paper or plastic. The grid was placed over the region whose area was to be determined. The following instructions were given to pupils:

1. Completely shade every unit of which at least partly covers some of the surface of the region (see Fig. 2).

2. Count the number of units just shaded. (The answer is eighteen.)

3. Next, completely shade (using different kind of shading or a new grid of the same kind of the same

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[34] Ibid., p. 669.

[35] Ibid.

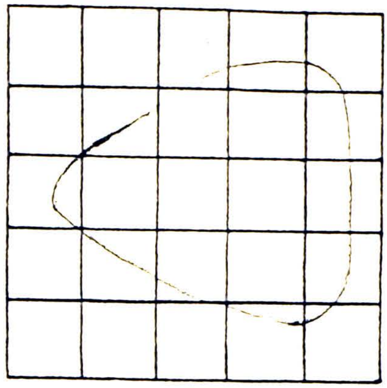

Figure 2

unit) every unit that lies entirely inside the region (see Fig. 3).

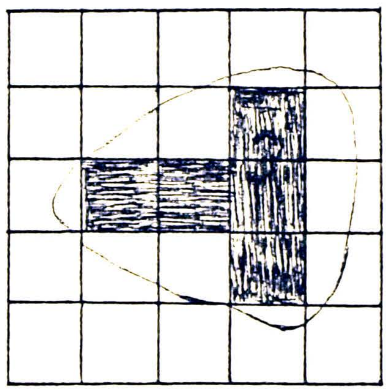

Figure 3

4. Count the number of units shaded this time. (The answer is 5.)

5. Now, put together the information obtained in questions two and four and conclude the following:

$5 <$ area of the region < 18;
that is, the area of the
region is a number
between 5 and 18.[36]

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[36] *Ibid.*, pp. 669 - 671.

Strangman added that when the steps were repeated by using a smaller unit, it was possible to obtain a better approximation for the area. After the students had considerable practice of different regions, they began to generalize that the smaller the grid size, the better the approximation for the area of the region.[37]

Formula Development of Area

The concept of area should be informally introduced as counting activities. Paige et al. maintained that these first activities involved counting the total number of units that cover a region.[38]

Paige et al. acknowledged that geoboards, graph paper, and dotter paper served as square units and were useful in helping the child develop the concept of area and discover formulas. Children who used rectangular arrays for solving multiplication problems readily discovered the formula of a rectangle. Once students realized that the diagonal of a rectangle cut the region into two congruent parts, they were able to find the area of a triangle. Paige et al. also stressed that the formula for the area of a triangle was very useful in developing the area of more complex figures.[39]

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[37] Ibid.

[38] Paige, op.cit., p. 231.

[39] Ibid.

Thomas contended that once students were familiar with three methods for finding area--(1) count unit squares; (2) slice-and-shuffle to convert the original figure to another figure having an area easy to determine; or (3) combine two congruent figures to form a larger, different figure having an area easy to compute--they had enough background to tackle trapezoid-area problems without any help from the teacher. Also, students discovered that a variety of approaches could be used in solving one problem, and it showed students how a procedure that fitted in a specific instance could be generalized (formula discovered).[40]

MISCONCEPTIONS ABOUT AREA MEASUREMENT

A case study designed to investigate the child's conception of area was conducted by Hirstein, Lamb and Osborne. In the study, children were interviewed through a series of tests. They were shown a display region, then were asked to draw a vertical line on a comparison strip to make a region with the same area as the display region (see Figure 4).

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[40] Diane Thomas, "Geometry in the Middle School: Problems with Trapezoids," The Arithmetic Teacher, XXVI (February, 1979), p. 20.

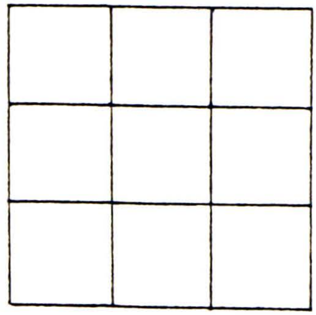

Figure 4

Display Figure

The questions were presented in three settings: no markings on the regions (no grid), the regions marked with linear units (indicated grid), and the regions marked with square area units. Figure 5 illustrated the settings.[41]

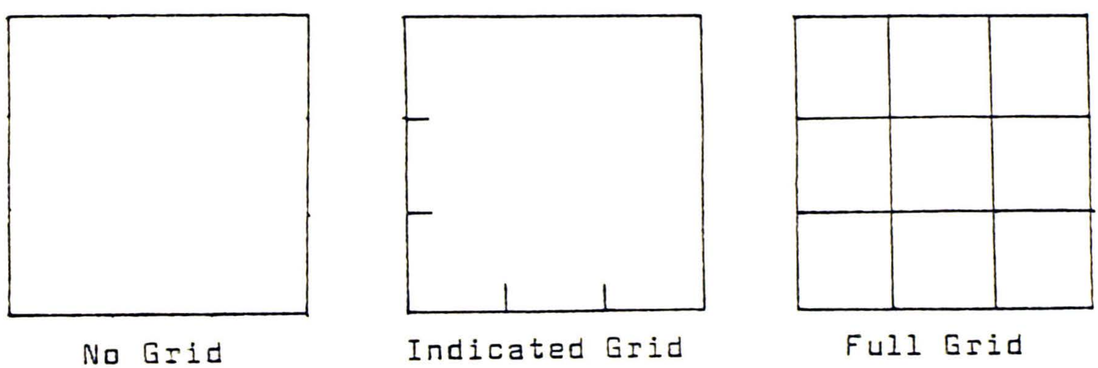

Figure 5

Perceptual Settings

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[41]Charles E. Lamb and James J. Hirstein, "Point Counting in Measurement - or - What is the Unit?" ed. Robert Todd and Thomas Teates, "Measurement Corner," <u>School Science and Mathematics</u>, LXXIX (February, 1979), p. 169.

Hirstein et al. furthered added that the interview consisted of eighteen items. Upon completion of each item, children were asked to give an explanation on how they had "figured out" where to put the line. All actions of the children were observed by the interviewer.[42]

According to Hirstein et al., the general performance of the children indicated that all children had some concept of area. Older children did better than younger, more experienced children. Also, older children were more likely to exhibit a feeling for area as a number indicating the space-filling character of measurement.[43]

Hirstein et al. disclosed that five misconceptions about area were observed in the behavior of several of the children. They were as follows:

1. Used the length of one dimension to make area judgement.

2. Used primitive compensation methods.

3. Point counted for area. (Counted points instead of units.)

4. Counted around the corners.

5. Point-counted linear units. (Counted the wrong entities on the edges of the figures.)[44]

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[42] Hirstein, Lamb, and Osborne, op.cit., p. 10.

[43] Ibid.

[44] Ibid., pp. 12 - 16.

Having reviewed the study of Hirstein, Lamb and Osborne, Todd and Teates concluded that misconceptions about linear measurement were often carried over to area measurement. Children did occasionally count points rather than area units. Therefore, he urged teachers to be sure that the unit concept be introduced in a thorough and meaningful manner. Also, teachers needed to be aware that children's approaches to area problems were sometimes based on misperceptions of linear units.[45]

Many children and adults had trouble dealing with area and perimeter. They often thought that two patterns with the same perimeter, though different in shape, must have had the same area and vice versa. Walter expressed concern over the two concepts involved. She suggested activities that would give students opportunities to discover the concepts of (1) fixed perimeter and changing area; and (2) fixed area and changing perimeter. The suggested activities were as follows:

1. Give students graph paper. Have them make shapes with perimeter of twenty-four units that have an area of any number of square units between zero and forty-five. Investigate several such problems using different perimeters. Have students give the shape of the largest area you can get from a fixed perimeter.

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[45]Lamb and Hirstein, op.cit., p. 171.

2. Turn the problem around. Suppose the shape has an area of twenty. Then find the perimeter of each shape. Have students decide if two different shapes with the same area have the same perimeter. Draw other shapes with an area of forty-eight square units.[46]

Walter discovered after several such exercises, children no longer felt that shapes with the same perimeter always had the same area--or that shapes with the same area always had the same perimeter.[47]

Laboratory experiences provided an opportunity for learners to find the area of selected geometric figures in a way which illustrated what area was. Brougher advocated using one-inch square pieces of paper. The student was given a worksheet, with instructions to cover the four geometric figures with the one-inch squares. (Experience has shown that one-inch squares of paper were easier for pupils to handle.) Upon completion of the activity, the students were able to find the area of selected geometric figures by finding how many one-inch squares were needed to exactly cover the figure.[48]

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[46]Marion Walter, "A Common Misconception about Area," The Arithmetic Teacher, XVII (April, 1970), pp. 287 - 288.

[47]Ibid.

[48]Brougher, op.cit., pp. 282 - 283.

SUMMARY

A review of authors in the field revealed that area was one of the hardest concepts in mathematics for children to understand. Also, it was important for a child to acquire because it was the most commonly used domain of measure in everyday life.

Mathematics educators have become concerned because students do not have an adequate understanding of area. They pointed out that area was being taught with a too rapid progress to formula-based computation and too many short cuts were being employed. Children needed to first understand that area was the surface coverage of an object and the units of measure were in terms of square inch, square foot, etc.

Laboratory activities needed to be designed to introduce the topic of area because understanding of the measures actually came from actual manipulation of concrete materials. The methods of filling and covering with unit squares and partitioning and recombining provided the child with subconcepts that were necessary to understand the actual concept of area. Manipulative experiences gained in dealing with these laboratory exercises built intuitive understanding of the concept of area. Pictorial representation of area problems had more meaning after having had these experiences.

Educators also expressed concern over some pitfalls and misconceptions children exhibited while working with area. They contended that it was important to watch students work in area problem settings in order to diagnose misconceptions. Once the misconception was identified, corrective activities should be designed for the children. The remedial activities used should stress two fundamental concepts. One is the idea of a unit and its space-covering characteristic. The second concept is that area remains constant when a figure is partitioned and recombined.

Although methods varied and objectives were worded differently, there was basic agreement that once students were familiar with three methods for finding area--(1) count unit squares; (2) slice-and-shuffle to convert original figure to another figure having the same area; or (3) combine two congruent figures to form a larger, different figure having an area easy to compute--they had enough background to tackle more complicated problems and discover formulas for themselves.

Chapter 3

COLLECTION, PRESENTATION, AND INTERPRETATION OF DATA

The author presented in this chapter a discussion of the procedure used for collecting data, a description of the instrument (test) used, and the summary.

The Stewart County Schools were chosen for this study for the convenience of the author. Also, these were chosen because of their reputation for cooperation with the University in students' research projects.

The author decided that eighth grade students from all three elementary schools in Stewart County were needed for the study. This was decided in order to involve as many students (academic backgrounds) as possible in the study.

PROCEDURE FOR SECURING SCHOOL COOPERATION

Prior to the study, permission was obtained from Mr. Van Riggins, Superintendent of Schools for Stewart County, to involve in the study all eighth grade students (Year 1979-1980) from the three elementary schools in Stewart County (see Appendix A, page 48). The three schools involved were North Stewart Elementary, W. T. Thomas Elementary, and Dover Elementary.

The author also thoroughly discussed the study with Mr. Bill C. Cherry, Supervisor of Schools, to secure his cooperation in the study. The principals of the three elementary schools were informed of the study by Mr. Cherry. The study was explained and permission granted for the writer to visit each school and administer the test used in the study.

Mr. Cherry and the principals allowed the author and the eighth grade mathematics teachers of North Stewart and W. T. Thomas to decide on the date and time of the testing. (The author was the eighth grade math teacher at Dover.) The author asked the two mathematics teachers from North Stewart and W. T. Thomas not to "help" the students with the concept of area prior to the testing period.

DESCRIPTION OF THE INSTRUMENT

With the help of Dr. Ernest Woodward, the researcher prepared a test on the concept of area in mathematics (see Appendix B, page 51). The test included seven activities. The concept in each activity was as follows:

1. Activity I - Perimeter constant, area changes.
2. Activity II - Area and perimeter of polygons with pictorial representation by use of formulas.
3. Activity III - Area and perimeter of rectangles without pictorial representation by use of formula.

4. Activities IV and V - Unit approach to determine area and perimeter of polygons.

5. Activity VI - Unit approach for area and perimeter. (Constant perimeter and changing area.) Also, the closer the polygon gets to being a square, the greater the area.

6. Activity VII - Perimeter constant, area changes.

PROCEDURES FOR COLLECTION OF DATA

After completion of the test (See Appendix B, page 51.), the author proceeded in the following manner:

1. Administered the test to a select group (five students) of top seventh grade students. This was done in order to determine if the test had any "rough spots" or if the directions were sufficient. The pilot test was quite satisfactory; therefore, no revision in the test was necessary.

2. Administered the test to all eighth grade students in Stewart County. (Students absent from school on the day of the test were not given the test at another time.)

3. The researcher and her aide administered all the tests in order to standardize the procedure. The regular classroom teachers did not stay in the room at the time of the test. The researcher did not ask them to leave; they left on their own accord.

ADMINISTRATION OF TEST

The author administered the test about two months after the beginning of school on three consecutive days at the same time each day. The test was administered in parts. The students were first given Activity I (see Appendix B, page 51). They were instructed not to put their names on any of the test sheets. The test giver (the author) explained and drew on the chalkboard the notation for a right angle. The students were instructed to complete the activities individually on Activity I. All answers were put directly on the test sheet in the places indicated.

The students were told not to worry if they did not know an answer. They were encouraged to do their best. Also, they were encouraged to answer each question, since anything left unanswered would be scored as a would response. When they finished, they placed their papers facedown on the desk. There was no time limit on these activities, but students were encouraged to finish in a reasonable length of time. When all had finished, the test giver collected the sheets and gave the students Part II, Activities II and III. Basically, the same instructions were used for Part II as in Part I. Again, the students were encouraged to finish in a reasonable length of time. When all had finished, the sheets were collected.

Next, students were given Part III, Activities IV, V and VI. Again, they were asked not to put their names on the sheets. Also, all answers were written directly on the test sheets in the provided places.

The test giver read and explained the directions of the three Activities. Examples to explain the unit and square unit were shown on the chalkboard. Students were encouraged to work without conversing with one another. When all had finished the activities, the sheets were collected.

Activity VII, the last part of the test, which was the same as Activity I, was given to the students. (Since Activities IV, V, and VI were originally designed as a learning experience, students, after having done these activities, should have had a better understanding of the concept of area and have known the correct answer to Activity VII.)

The author administered the test at Dover Elementary first. No changes were made in it thereafter.

After the three eighth grade classes in the county had taken the test, the author did the following:

1. Scored and evaluated all the tests.

2. Compiled data, drew conclusions, and made recommendations.

3. Sent the Supervisor of Schools for Stewart County a copy of the evaluation of the results of the tests along with recommendations of various methods to be used in teaching the concept area in mathematics.

EVALUATION OF TEST RESULTS

After scoring the tests, the author analyzed the results and recorded them in the following manner:

1. Computed the percentage of right and wrong answers for each problem.

2. Recorded the results on a chart.

3. Computed concept difficulty and showed results on a graph in percentage of right and wrong responses for each item (or area). The author recorded the results of the tests on a chart (see Appendix C, page 59) and a table (see Appendix C, page 59). The results were used (plus a careful look at each individual test) to analyze indicators and make generalizations.

4. Recommendations were made for improving the teaching of area in Grades 4 - 6 in the Stewart County School System.

A breakdown of the analysis of the activities was as follows:

<u>Activity I</u>

Since only twenty-three percent of the students responded to this activity correctly, it was evident that the majority did not understand the concept that the area can change though perimeter remains constant. Over half of the students thought the gardens were all the same size since the amount of fencing was the same. The author gave the activity to several elementary teachers and to some other friends, and a majority of them gave the wrong response. Therefore, it is the opinion of the writer that the average person does not understand the concept involved in Activity I.

Activities II and III

There was very little difference in the percent of correct responses for finding the area and perimeter of figures in these activities. Activity II had the pictorial representation of the figures, but they did not seem to help. This only verifies what some educators have uncovered in recent studies. They found that if a child does not understand the concept of area, pictorial representation with written explanations do not help him find the area or perimeter.

The triangle and parallelogram were the only figures that had a vast difference in the correct responses for perimeter and area. Only eight percent of the students correctly found the area of the triangle; whereas, thirty-six percent correctly found the perimeter. It was evident that the students did not realize the area relationship between a right triangle and a rectangle. Nor, did they realize the relationship that existed between the area of the rectangle and the parallelogram because only five percent of the students correctly found the area of the parallelogram.

The students' concept of perimeter was about the same for all seven of the figures. When scoring Activity

II, the author did not consider whether or not the student correctly identified the square units in the answer.

Activities IV, V, and VI

Activities IV, V, and VI were originally designed to be learning experience as well as a test for the students. The students seemed to enjoy this part of the test the most. Activity IV had the highest percentage of correct responses. This involved the unit approach to perimeter. Activity V had the next highest percentage of correct responses. It involved the unit approach to area. Their score was excellent on the area of rectangles using the unit-counting approach, but fell greatly when they tried to compute by unit-counting the area of the triangle and the irregular polygon. It appeared that the one-half units were difficult for them to "count."

Activity VII

Activity VII was the same as Activity I. The authors of the test had hoped the students would learn from Activities IV, V, and VI and would understand the concept involved in Activity VII and give the correct answer. This did not happen. There was only about a six percent increase in the number of correct answers. Some who had gotten it right in Activity I missed it in Activity VII. Others who had missed changed their answers, but still did not have the right answer.

The author observed that the three activities were certainly not enough to help students who were already deficient in the concept which involved a constant perimeter and changing area. Only one student of those tested openly showed he had been enlightened by the learning experiences in Activities IV, V, and VI. After having done these activities, he openly expressed that he felt so "dumb" because he had missed Activity I. The author carefully watched the student turn to the final activity. The student looked up, gave a big smile, and marked the correct choice.

As an overview of the actual testing, the writer thought that the attitudes of the students involved were very good. When the test was over, many students were apologetic because they had "forgotten how" to find the area of geometric figures.

Chapter 4

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

SUMMARY

The purpose of the study was to determine if beginning eighth grade students (Year 1979 - 1980) in Stewart County understood the concept of area in mathematics. A review of the related literature revealed that educators have expressed much concern about students' lack of understanding and their misunderstanding of the concept of area in mathematics. It was the author's opinion that the study would reveal "if" and to what extent students understood the concept of area in mathematics.

The instrument used to measure the students understanding of the concept of area was a teacher-made test designed by Dr. Woodward and the author. The activities in the test involved five important concepts of area.

The study was limited in time, sample, and scope and was confined to concepts measured by the teacher-made test.

Permission to conduct a study of eighth grade students in the Stewart County School System was granted by the Superintendent of Schools. The author discussed the study with Mr. Bill C. Cherry, Supervisor of Schools, to secure his cooperation in the study. He met with the principals of the three schools involved and informed them of the study. The principals then informed the eighth grade mathematics teachers of the study. (Only two other teachers were involved since the author was one of the eighth grade mathematics teachers in Stewart County.) The results of the test were evaluated by the writer and were interpreted as yielding the following conclusions.

CONCLUSIONS

As a result of the study, the author concluded that:

1. Very few eighth grade students understood the concept of area in mathematics.

2. It appeared that students had not had experiences with the subconcepts of area.

3. A formula-based approach had been presented to the students too early instead of a proper unit-counting, figure-filling approach.

4. The concept of area, though included in their textbooks for each grade, was not always included in the math program for the student.

RECOMMENDATIONS

As a result of reading related literature, and evaluating the results of the teacher-made test administered to eighth grade students in Stewart County Schools, the author made the following recommendations:

1. The concepts of area should be included in the mathematics curriculum in grades K - 8.

2. A laboratory approach should be used in teaching the concepts.

3. Suggested methods and activities for teaching the concept of area are as follows:

a. The student needs to have experience in primitive subconcepts in order to develop an intuitive feel for area.

b. The first activities in "area" should involve counting the total number of units that cover a region. One-inch paper squares are easier to handle.

c. Once the concept of a unit is present, the next is the iteration of the units to assign a number to a given region. Graph paper, geoboards, and dotter paper can serve as square units.

d. Through instruction from the teacher and manipulation of figures, the student must be made to conceive of a region as a union of subregions connected at the boundaries.

e. The student should partition and recombine (slice-and-shuffle) to convert the original figure into another figure having an area easy to determine.

f. Tangrams should be used to introduce the concept that the shape of the figure can change without changing area.

g. Combine two congruent figures to form a larger, different figure having an area easy to compute.

h. Emphasis should be placed not on formulas, but on the understanding that in area that you are working with square units.

i. The teacher should follow up activities with problem-solving situations that are not only interesting but also lead students to focus on the attributes of area.

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APPENDICES

Appendix A

CORRESPONDENCE

Stewart County Board of Education

AN RIGGINS, SUPT.
(615) 232-5176

P.O. BOX 40
DOVER, TENNESSEE 37058

July 13, 1979

Mrs. Frances Byrd
P. O. Box 7
Dover, Tennessee 37058

Dear Mrs. Byrd:

The Stewart County Board of Education grants permission for you to test Stewart County 8th grade students on their concept of area.

This is to be done during the fall semester of 1979.

Yours truly,

Van Riggins

Van Riggins
Superintendent of Schools

VR/mcs

Stewart County Board of Education

VAN RIGGINS, SUPT.
(615) 232-5176

P.O. BOX 40
DOVER, TENNESSEE 37058

TO: Stewart County Elementary Principals

FROM: Bill C. Cherry, Supervisor BC

DATE: November 9, 1979

Frances Byrd has permission to administer tests to eighth grade students in Stewart County's schools. Mrs. Byrd will contact all principals and finalize details such as, dates, times, etc.

Your cooperation in this matter is appreciated.

P. O. Box 7
Dover, TN 37058

Mr. Bill C. Cherry, Supervisor
Stewart County Schools
P. O. Box
Dover, TN 37058

Dear Mr. Cherry:

Having completed my study on to what extent the eighth grade students (Year 1979-80) in Stewart County understood the concept of area, I have come to the following conclusions:

1. Very few eighth grade students understood the concept of area in mathematics.

2. It appeared that students had not had experiences with the subconcepts of area.

3. A formula-based approach had been presented to the students too early instead of a proper unit-counting, figure-filling approach.

4. The concept of area, though included in their textbooks for each grade, was not always included in the math program for the student.

Therefore, I present you a list of recommendations for teaching area to students in grades K-8. Also, I enclose a copy of the test results and my evaluation of the tests.

If it meets with your approval, I would like for you to give a copy of these recommendations to each teacher in Stewart County who teaches mathematics in elementary or junior high.

Thank you for your cooperation in this study.

Sincerely yours,

Frances Byrd

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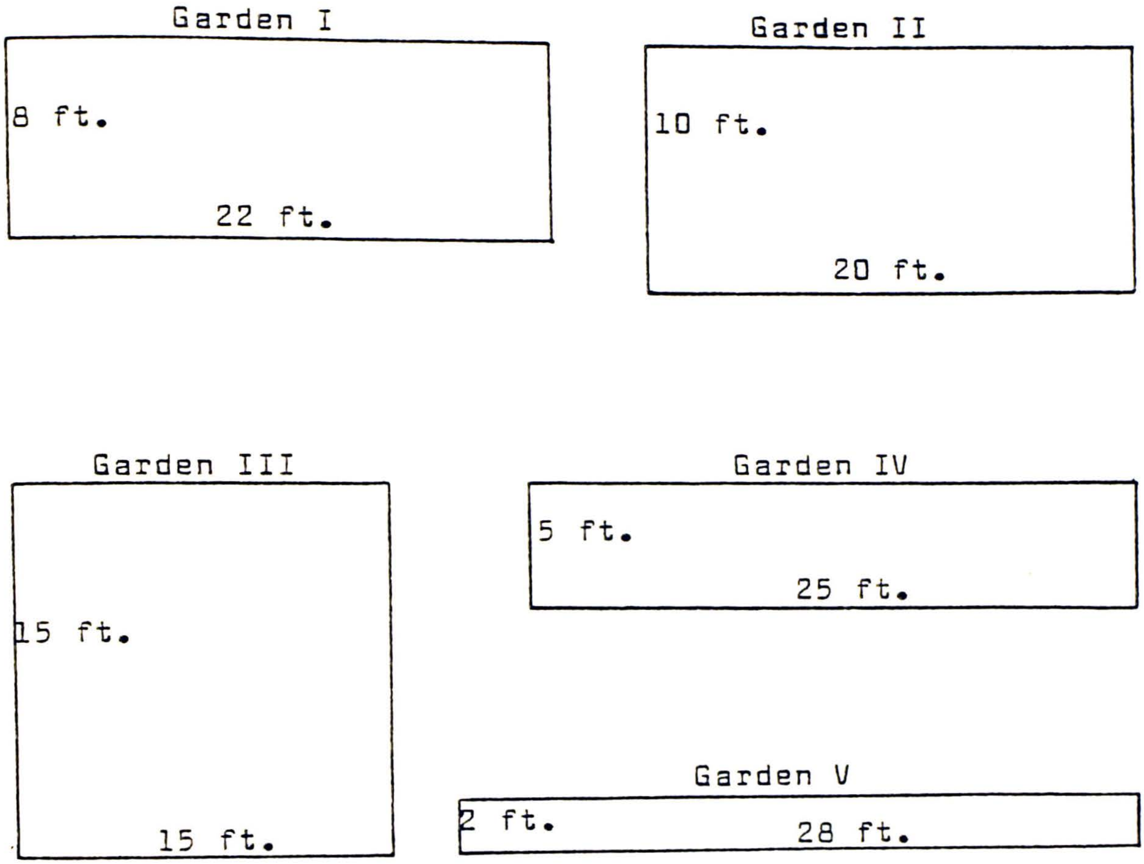
Appendix B

TEACHER MADE TEST OF AREA IN MATHEMATICS

Activity I

Mr. Young had 60 feet of fencing available to enclose a garden. He wanted the garden to be rectangular in shape. Also, he wanted to have the largest possible garden area. He drew a picture of several possibilities for the garden, each with a perimeter of 60 feet. These drawings are pictured below.



Consider Mr. Young's drawings of the garden plots.

Check the statement below that he found to be true.

______1. Garden I is the biggest garden.

______2. Garden II is the biggest garden.

______3. Garden III is the biggest garden.

______4. Garden IV is the biggest garden.

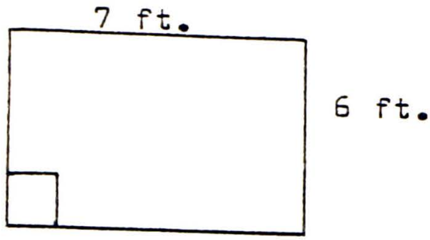
______5. Garden V is the biggest garden.

______6. The gardens are all the same size.

Activity II

Find the area and perimeter of each geometric figure.

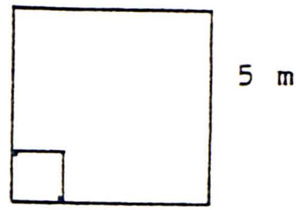
Rectangle



The area is ______.

The perimeter is ______.

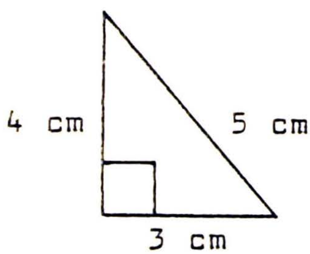
Square



The area is ______.

The perimeter is ______.

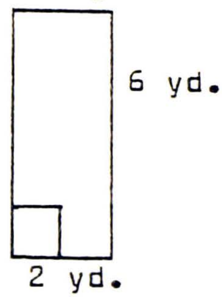
Triangle



The area is ______.

The perimeter is ______.

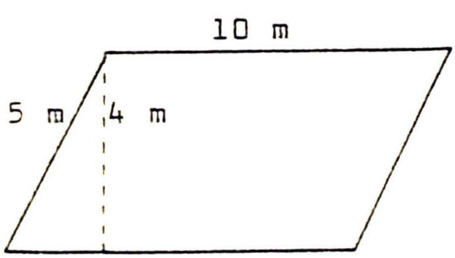
Rectangle



The area is ______.

The perimeter is ______.

Parallelogram



Activity III

Find the area and perimeter of each.

a. Rectangle - Length of 7 cm, width of 9 cm.

The area of the rectangle is ______ cm^2.

The perimeter of the rectangle is ______ cm.

b. Square - Side is 7 m.

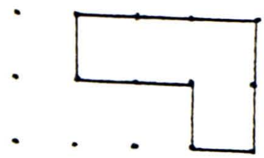
The area of the square is ______ m^2.

The perimeter of the square is ______ m.

Activity IV

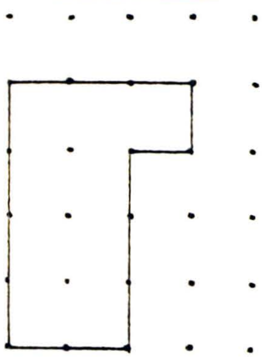
Below you will find some arrays of dots. The dots have been connected to form geometric figures. Two adjacent dots which have been connected vertically ( ) or horizonally ( ) is one unit of measure.

Example: In the figure at the right, the measure of the distance around it is 10 units.



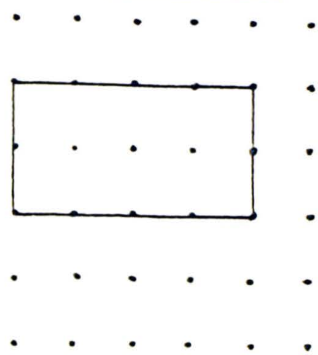
Determine the number of units around the sides of each of the following figures.

Figure I



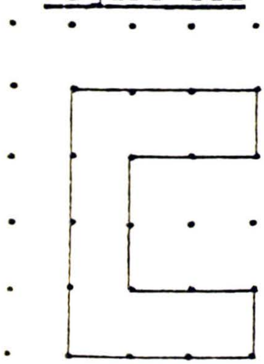
The perimeter is ____ units.

Figure II



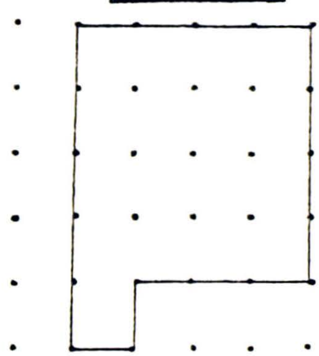
The perimeter is ____ units.

Figure III



The perimeter is ____ units.

Figure IV

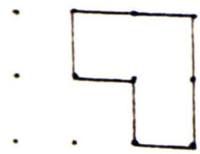


The perimeter is ____ units.

Activity V

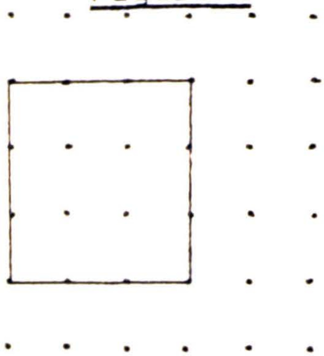
The arrays of dots below have been connected to form geometric figures. The measure of the surface within 4 dots is one square unit:

Example: In the figure at the right, the area is 3 square units.



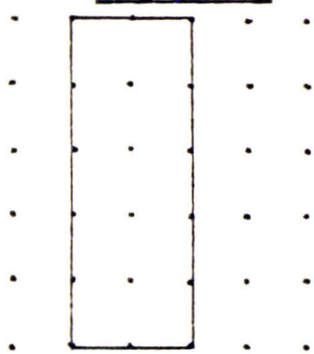
Determine the number of units in each of the following figures.

Figure I



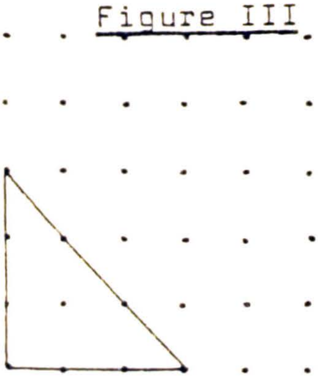
The area is _____ square units.

Figure II



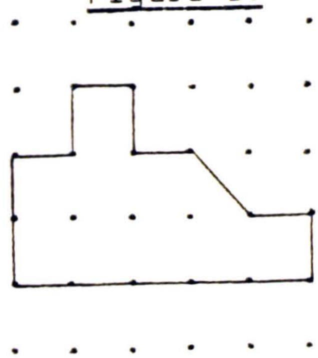
The area is _____ square units.

Figure III



The area is _____ square units.

Figure IV



The area is _____ square units.

Activity VI

On each section of dots, draw a different picture of a rectangle with a perimeter of 20. (There are five possible rectangles that can be drawn with a perimeter of 20.) After drawing the rectangles, complete the table on the following page. Figure I has been drawn for you. Draw the others.

Figure I

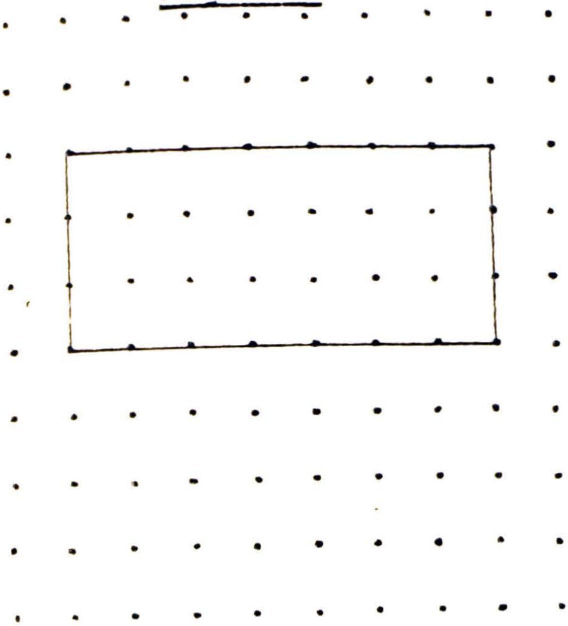
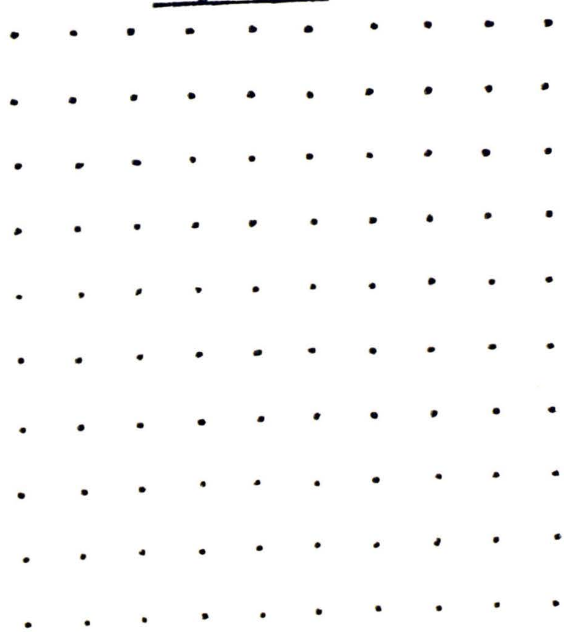


Figure II

Figure III

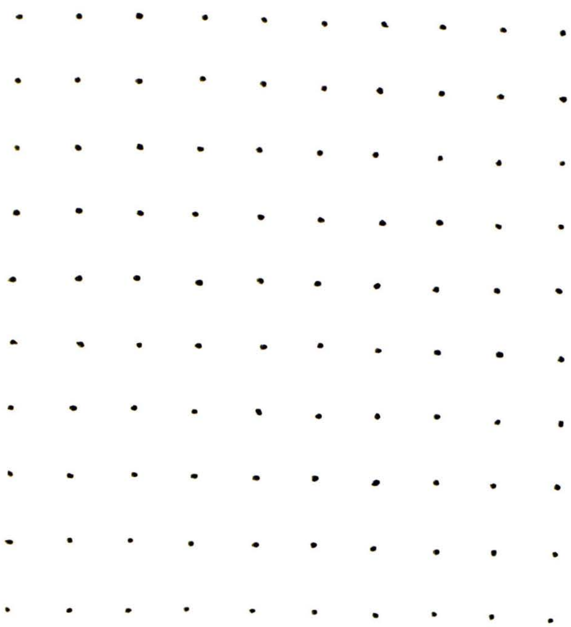
Figure IV



(continued on next page)

Activity VI (continued)

Figure V



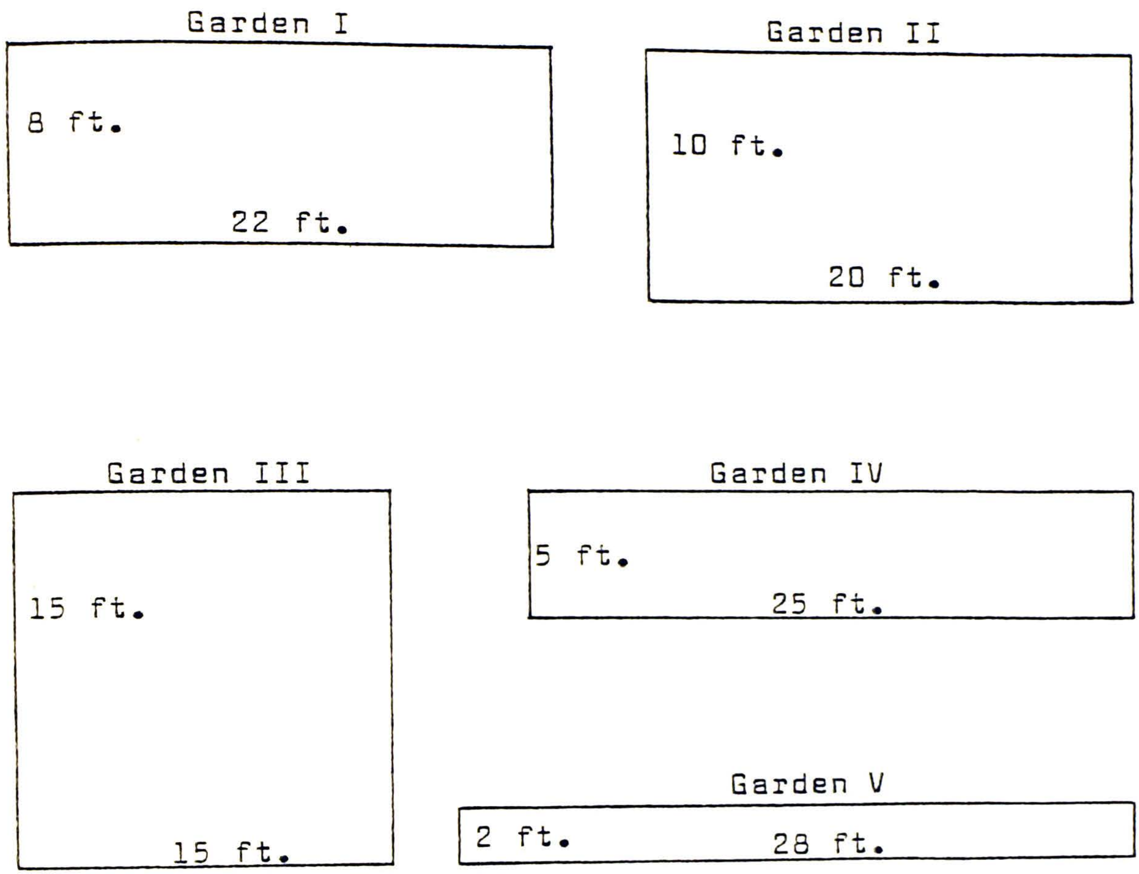
Consider the rectangles you have drawn to complete the following table.

	Size	Area
Figure I	3 x 7	21
Figure II		
Figure III		
Figure IV		
Figure V		

Which shape gave the greatest area? ______

Activity VII

Mr. Young had 60 feet of fencing available to enclose a garden. He wanted the garden to be rectangular in shape. Also, he wanted to have the largest possible garden area. He drew a picture of several possibilities for the garden, each with a perimeter of 60 feet. These drawings are pictured below.



Consider Mr. Young's drawings of the garden plots.

Check the statement below that he found to be true.

______1. Garden I is the biggest garden.

______2. Garden II is the biggest garden.

______3. Garden III is the biggest garden.

______4. Garden IV is the biggest garden.

______5. Garden V is the biggest garden.

______6. The gardens are all the same size.

Appendix C

PERCENTAGES OF RESPONSES BY ACTIVITIES

Table I

Percentage of Responses by Activities

	% Right Answers	% Wrong Answers	Total %
Activity I	23	77	100*
Activity II			
Rectangle-Area	25	75	
-Perimeter	33	67	
Square-Area	22	78	
-Perimeter	33	67	
Triangle-Area	8	92	
-Perimeter	36	64	
Rectangle-Area	28	72	
-Perimeter	29	71	
Parallelogram-Area	5	95	
-Perimeter	29	71	
Activity III			
Rectangle-Area	23	77	
-Perimeter	24	76	
Square-Area	16	84	
-Perimeter	28	72	
Activity IV			
Figure 1	87	13	
Figure 2	91	9	
Figure 3	84	16	
Figure 4	86	14	
Activity V			
Figure 1	83	17	
Figure 2	86	14	
Figure 3	26	74	
Figure 4	27	73	

* Each entry hereafter totals 100%

Table I (continued)

	% Right Answers	% Wrong Answers	Total % *
Activity VI			
4 figures	21		
3 figures	12		
2 figures	13		
1 figure	25		
0 figures	29		
Table			
Figure 2	33	67	
Figure 3	39	61	
Figure 4	40	60	
Figure 5	26	74	
Shape of Greatest Area	19	81	
Activity VII	29	71	

* Each entry hereafter totals 100%

Note: Numbers were rounded off for convenience where applicable.

Graph 1

Correct Responses for Each Concept*

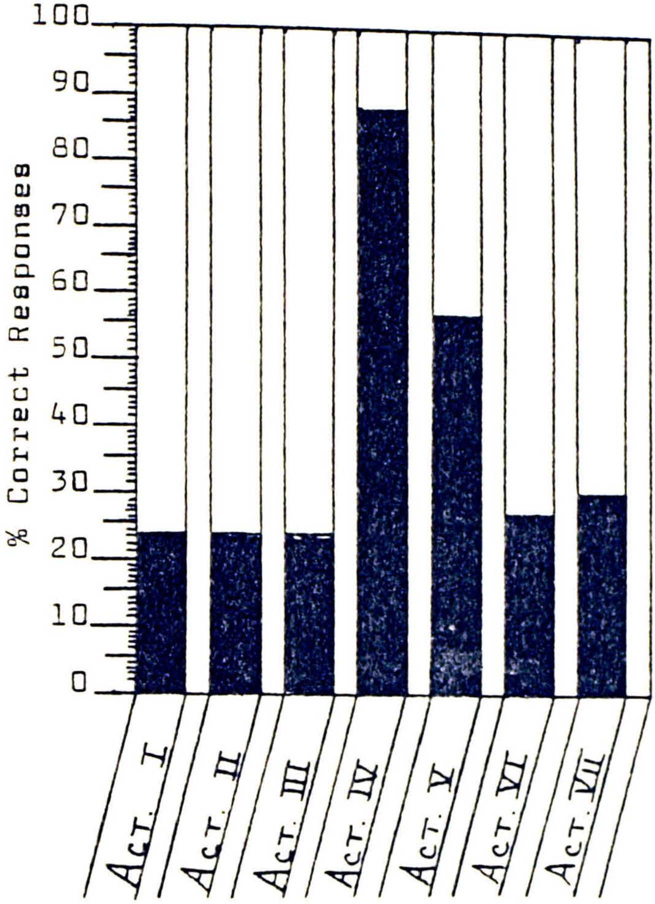


Figure 1 - *Concepts Included in Activities

Activity I - Perimeter constant, area changes.

Activity II - Area and perimeter of polygons with pictorial representations by use of formulas.

Activity III - Area and perimeter of rectangles without pictorial representation by use of formula.

Activities IV and V - Unit approach to determine area and perimeter of polygons.

Activity VI - Unit approach for area and perimeter of rectangles. Constant perimeter and changing area.

Graph 1 (continued)

Activity VI (continued)

Square shape provides greatest area. (Closer the figure gets to being a square, the greater the area.)

Activity VII - Perimeter remains constant, area changes.