A STUDY OF PREDICTION OF ACHIEVEMENT IN CALCULUS

BY

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A STUDY OF PREDICTION OF ACHIEVEMENT IN CALCULUS

A Research Paper

Presented to

the Graduate Council of

Austin Peay State University

In Partial Fulfillment

of the Requirements for the Degree

Master of Arts

in Education

by

Betty Hurt Hester

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To the Graduate Council:

I am submitting herewith a Research Paper written by Betty Hurt Hester entitled "A Study of Prediction of Achievement in Calculus." I recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Arts in Education, with a major in Mathematics.

William G. Stokes
Major Professor

Accepted for the Council:

Wayne E. [illegible]
Dean of the Graduate School

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CHAPTER I

INTRODUCTION

The predicting of achievement in many subjects has been an important field of study for many years. In mathematics, it is especially important to be able to predict achievement in freshmen college calculus. Students who enter calculus as freshmen mathematics majors are generally required to have successfully completed four years of high school mathematics. This is usually not sufficient to predict that the students will do well in their first calculus course. It is very disheartening to a would-be mathematics major to fail his first college mathematics course. To avoid this problem, colleges may require that all entering mathematics majors take College Algebra or Trigonometry or both. However, this would cause many students to sit through a course which repeats material with which they are already familiar. Therefore, it is desirable to develop some means of predicting achievement in first quarter college calculus.

RELATED LITERATURE

Many attempts have been made to find good predictors of achievement in college calculus. Crosswhite, in a study at Ohio State University, experimented with fifteen different variables. His criterion of success was the final grade in calculus, and his criterion of achievement was the total score on the two tests given in the course. He found that high school recommendation was an "unreliable and inadequate substitute for objective data." Students who had had calculus and analytic geometry in high school did better than those who had not had it. "The score from the locally constructed placement test in mathematics was found to be the best single predictor examined."[1]

McKillip, in a study at the University of Virginia, supported the findings of Crosswhite. McKillip found that students who had had calculus in high school made significantly better grades in analytic geometry and calculus than they would have been expected to make if they had not had calculus in high school.[2]

Francis compared three tests to see which was the best predictor of achievement in Calculus I. He used the three tests and the achievement in Calculus I as predictors of achievement for Calculus II. The tests were the Mathematics Advanced Placement Tests In Algebra And Trigonometry and the Quantitative School And College Ability Test. The measure of achievement in Calculus I was the Calculus I Achievement Examination. The best predictor of achievement in Calculus I was the Mathematics Advanced Placement test in Algebra. The best predictor in Calculus II was the Calculus I Achievement Examination.[3]

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[1] F. Joe Crosswhite, "Procedures For Admission With Advanced Standing In Mathematics At The Ohio State University," Dissertation Abstracts, 25:6427, May, 1965.

[2] William David McKillip, "The Effects of Secondary School Analytic Geometry and Calculus on Students' First Semester Calculus Grades at the University of Virginia," Dissertation Abstracts, 26:5920-5921, April, 1966.

[3] Richard Lee Francis, "A Study of the Value of Selected Test Scores for Predicting Success In Analytic Geometry and Calculus," Dissertation Abstracts, 26:7166, June, 1966.

Ryan also used three tests as predictors of achievement. Achievement was measured by the final grades in the first three quarters of calculus and the first two quarters of physics. The tests used were the MNL Mathematics Achievement Test, Form GH, the Institute of Technology Mathematics Test, Form L, and the Minnesota Scholastic Ability Test. The student population was divided into students who had had modern (SMSG) mathematics and those students who had had conventional instruction. The results showed that the IT Mathematics Test was the best predictor of achievement for those students who had conventional instruction. The best predictor for those students who had modern mathematical instruction was the MNL Mathematics Achievement Test.[4]

Wampler used eleven tests representing eight aptitude factors to find the best linear combination of predictors of achievement in calculus. The resulting combination represented the following ability factors: "induction, general reasoning, syllogistic reasoning, verbal comprehension, and visualization." The best single predictor in this study was the Wide Range Vocabulary Test.[5]

Howlett used eight predictor variables for predicting success in the first calculus course at Austin Peay State College in 1966. The variables were:

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[4] James J. Ryan, "Previous Instructional Program as a Moderator of the Predictive Validity of College Entrance Tests in Mathematics," Educational and Psychological Measurement, 28:937-941, Autumn, 1968.

[5] Joe F. Wampler, "Prediction of Achievement in College Mathematics," Mathematics Teacher, 59:364-369, April, 1966.

X_1: grade point average in high school mathematics

X_2: high school grade point average

X_3: class standing in high school

X_4: ACT English score

X_5: ACT mathematics score

X_6: ACT social studies score

X_7: ACT science score

X_8: ACT composite score

The criterion of achievement in this study was the final grade in calculus. Howlett found the correlation between this criterion and each variable. He found the intercorrelations of all variables. He then combined the independent variables using a multiple regression procedure. The best single predictor was the high school grade point average in mathematics. Class standing in high school, ACT mathematics score, and the ACT composite score were the next best predictors. The best combination of predictors was the grade point average in high school mathematics and the ACT composite score.[6]

THE PROBLEM

The purpose of this study was to determine the effectiveness of a locally made test for predicting achievement in Mathematics 201 at Austin Peay State University as compared with variables established previously as good predictors of achievement in Mathematics 201 at Austin Peay State

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[6] John Leo Howlett, "A Study of Predicting Achievement in Analytical Geometry and Calculus," (unpublished Master's Thesis, Austin Peay State College, 1966).

College. The criteria for selecting the variables used in this study were: (1) the variables were shown to be correlated with the grade in Mathematics 201 at the one percent level of confidence in Howlett's study and (2) the information for these variables was easily obtainable from students' records at Austin Peay State University. The variables chosen from Howlett's study are the high school grade point average, the ACT composite score and the ACT mathematics score. The mathematics grade point average was not used because the high school grade point average was more easily obtained from the student's records than the mathematics grade point average, and the high school grade point average and the high school mathematics grade point average had a correlation of .790635 according to Howlett's study.[7] The independent variables used in this study were:

X_1: grade on the locally made test

X_2: high school grade point average based on a four point system

X_3: ACT composite score

X_4: ACT mathematics score

The measurement of achievement in this study was the final grade in Mathematics 201. This grade is usually reported as a letter grade, but for statistical purposes, the letters were converted to numbers using the following system: A = 95, B = 85, C = 75, D = 65, and F = 50. These numbers were chosen because instructors generally use 90-100 as an

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[7] Ibid.

A, 80-89 as a B, 70-79 as a C, 60-69 as a D, and below 60 as an F. A number was selected from each category to represent that category.

LIMITATIONS OF THE STUDY

This study was limited by the sample, the geographical area covered, and the number of variables used. Only thirty-nine students were used in this study. It would have been desirable to have used all fifty-nine students who were enrolled in Mathematics 201 at the time of the study, but there was no way of obtaining all the necessary information for these students.

It would also have been desirable to study only students who were taking calculus as their first college mathematics course. This would have eliminated the intervening variable of other college work. Using only students who were taking calculus as their first college mathematics course was impossible due to the small number of students from the sample who were in this category.

This study was limited to students at Austin Peay State University. It did not attempt to include students from other institutions. Therefore the inclusions may not necessarily be valid at other institutions.

Only four independent variables were used in this study. Ideally all of the variables discussed in Chapter I that had a significant correlation with achievement in a first course in calculus would have been included in this study. However, the inclusion of so many variables was beyond the scope of this study.

CHAPTER II

PROCEDURES

DESIGN OF THE EXPERIMENT

All students enrolled in Mathematics 201 at Austin Peay State University in the Winter or Fall Quarters of 1970 were given a locally made test during on of the first meetings of the class. The high school grade point average, the ACT composite score and the ACT mathematics score were obtained from the permanent record of each of the students for which this information was available. The final grade in Mathematics 201 was obtained for each student. Correlations between each of the independent variables and the final grade in Mathematics 201 were made using a 1401 computer. These correlations were compared to determine the best single predictor of achievement in Mathematics 201.

THE LOCALLY MADE TEST

Developing the Locally Made Test

The locally made test was constructed so that every student would reasonably be expected to finish it in the fifty minutes allowed for the test. The first four chapters of the text[8] for the course were used to determine what concepts should be included in the test. This

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[8] Robert C. Fisher and Allen E. Ziebur, Calculus and Analytic Geometry (Englewood Cliffs, New Jersey:Prentice-Hall, Inc., 1965), pp. 1-96.

was to assure that the test had content validity. Upon examination of these four chapters, it was determined that students beginning this text need to be able to solve problems involving inequalities and absolute value, graphing, algebraic manipulation, and trigonometry. The locally made test was designed to cover these topics. The following table gives a breakdown of the test items according to the topics covered.

TABLE I

BREAKDOWN OF LOCALLY MADE TEST ACCORDING TO TOPICS COVERED BY ITEMS

Topic Covered	Item Numbers
Inequalities and Absolute Value	1,2,3,4,5
Graphing	7,13,15,17,19,20,21
Algebraic Manipulation	8,11,12,14,16,18,22,24
Trigonometry	6,9,10,23

The reliability of the test was determined using the split-half method. The correlation between the scores students made on the even numbered items and the scores students made on the odd numbered items was calculated. The Spearman-Brown formula[9] was then used to determine the reliability of the total test was .629.

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[9] J. P. Guilford, Fundamental Statistics in Psychology and Education (New York:McGraw-Hill Book Company, 1965), pp. 457-458.

Administering the Locally Made Test

The reason for the test was discussed during a regularly scheduled meeting of the calculus classes early in the quarter. At the following class meeting the test with instructions printed on the front was given to all students in the classes. The students were allowed fifty minutes to complete the test.

THE POPULATION

The population for this study consisted of all students who have taken Mathematics 201 at Austin Peay State University. The sample used consisted of students who were taking Mathematics 201 at Austin Peay State University in the Winter or Fall Quarters of 1970. The locally made test was given to all fifty-nine students enrolled in Mathematics 201 during the Winter or Fall of 1970. Five students were in the classes both quarters. Their second scores were not used because they had prior knowledge of the test when they took it the second time. A total of twenty students were eliminated from the study because all the needed information was not available for them. Three of these students withdrew from the classes. Four students were from foreign countries. Even if information had been available for these students, it would probably not have been comparable to the information available for other students in the study. ACT scores were not available for eight students, probably because they did not enter Austin Peay State University in the fall. It is not felt that this would cause a significant difference in the results of the study. The high school grade point averages were not available for five students. No explanation is given for this, but it is not felt that there was any particular pattern to the absence of these records. The remaining thirty-nine students were used in the study.

CHAPTER III

ANALYSIS OF DATA

The data were analyzed by the use of a 1401 computer. The program used was written in Fortran language by the experimenter. Correlations between each of the independent variables and the dependent variable were calculated. (Table 2). Note that the correlation of .4728 between the locally made test and the final calculus grade and the correlation of .6022 between the high school grade point averages and the final calculus grade were significant at the five percent level of confidence. The best single predictor of achievement in calculus was the high school grade point average. The grade on the locally made test was the next best predictor.

TABLE II

THE CORRELATIONS OF THE INDEPENDENT VARIABLES WITH THE DEPENDENT VARIABLE

Variable	Correlation With Final Grade
X_1: test grade	.4728
X_2: high school grade point average	.6022
X_3: ACT composite	.1825
X_4: ACT mathematics score	.1208

Note: Correlation at the five percent level of confidence is .3165.

The correlation between ACT composite score and the final calculus grade was .1825. This was not significant at the five percent level of confidence, but it is interesting to note that this correlation is higher than the correlation of .1208 between the ACT mathematics score and the final calculus grade. Since neither of these correlations are significant, ACT composite scores and ACT mathematics scores are (according to this study) of no significant value in predicting success in calculus. The relationship that existed between these two variables and the criterion variable could be attributed to chance.

Upon analysis of the raw data one finds that a student with a minimum score of 40 on the locally made test and a minimum high school grade point average of 2.20 made a grade of C or better in 26 out of 32 cases. It is interesting to note that only two students who made less than 40 on the locally made test or had a high school grade point average of less than 2.20 made a C or better in the calculus course.

CHAPTER IV

SUMMARY AND CONCLUSIONS

The purpose of this study was to determine the effectiveness of a locally made test in predicting achievement in Mathematics 201 at Austin Peay State University as compared with variables established previously as good predictors of achievement in first quarter calculus. The independent variables used in this study were:

X_1: grade on the locally made test

X_2: high school grade point average

X_3: ACT composite score

X_4: ACT mathematics score

The measure of achievement in Mathematics 201 was the final grade in that course.

The locally made test was constructed and given to all students in Mathematics 201 at Austin Peay State University during the Winter and Fall Quarters of 1970. The records of these students were examined to obtain their high school grade point averages, their ACT composite scores, and their ACT mathematics scores. The students' final grades were obtained from their instructors. Students were eliminated from the study when all necessary information was not available.

The data were analyzed by a 1401 computer using a program written in Fortran language by the experimenter. Correlations between each of the four independent variables and the final grade in Mathematics 201 were calculated. The best single predictor of achievement in calculus was the high school grade point average.

Analysis of the data revealed that a minimum high school grade point average of 2.20 and a minimum score of 40 on the locally made test proved to be a reliable predictor of a final calculus grade of C or better in 26 cases out of 32. Only two students who made below 40 on the locally made test or who had a high school grade point average of less than 2.20 made a C or better in the calculus course.

The results of this study could be used to place students in college mathematics courses. Students with a high school grade point average of 2.2 or more and a minimum score of 40 on the locally made test could be placed in a calculus class while students with lower scores on either criteria could be placed in some lower mathematics class. However, it is not recommended that the high school grade point average be used alone for placement of students in calculus because students with a good high school grade point average may not have the necessary preparation or desire to take calculus.

It is suggested that future studies investigate the relationship between ACT scores and the calculus final grade since the results of Howlett's study and the present study did not agree on that point. It is also suggested that another study be performed to determine variables that could be used to predict achievement for students taking calculus as their first mathematics course as compared to those variables for students who have had previous college mathematics courses.

TABLE III

RAW DATA

Grade On Locally Made Test	High School GPA	ACT Comp.	ACT Math	Final Calculus Grade
66	3.59	22	28	B - 85
75	2.48	15	14	A - 95
71	3.05	25	23	B - 85
46	3.06	26	30	D - 65
71	2.88	26	27	F - 50
57	2.75	27	23	B - 85
46	2.80	23	24	C - 75
55	1.42	28	28	F - 50
31	2.24	17	21	D - 65
41	2.85	23	26	C - 75
61	2.61	23	25	F - 50
51	1.28	25	29	F - 50
16	2.22	15	17	F - 50
46	3.97	23	22	B - 85
81	4.00	28	30	A - 95
76	3.21	25	29	A - 95
32	2.92	20	20	D - 65
86	4.00	30	30	A - 95
21	2.97	26	28	C - 75
61	3.90	26	28	A - 95
51	2.22	25	27	B - 85
51	3.38	23	20	D - 65
66	3.75	24	29	A - 95
51	3.07	24	28	C - 75
76	3.39	25	28	A - 95
51	2.80	19	20	D - 65
61	3.69	23	25	A - 95
61	2.90	26	27	B - 85
41	3.46	15	24	B - 85
46	3.20	24	21	C - 75
41	2.56	20	18	A - 95
56	3.44	21	24	D - 65
71	3.54	28	29	A - 95
57	2.97	26	24	A - 95
24	3.70	26	21	C - 75
46	3.47	26	27	A - 95
51	3.80	27	31	C - 75
41	2.54	22	23	F - 50
71	3.54	21	20	A - 95

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APPENDIX

Scoring Formula
4R-W

CALCULUS 201

Pre-Test

Name ____________

Date ____________

List the mathematics courses you had in high school. After each, give the number of years you have studied each. (Semester = 1/2 year -- Quarter = 1/3 year)

____________________ ____________________

____________________ ____________________

____________________ ____________________

____________________ ____________________

____________________ ____________________

Have you had any college mathematics courses? ________ If so, list them and the number of years you have studied each. (Semester = 1/2 year -- Quarter = 1/3 year)

____________________ ____________________

____________________ ____________________

____________________ ____________________

____________________ ____________________

Directions: You have fifty minutes to work on the test. Each problem on the test is followed by five possible answers. Circle the correct answer. Do not spend too much time on one problem. Answer the easier ones first then go back to the hard ones. Answer questions even when you are not perfectly sure of the answers, but avoid wild guessing.

1. Which of the following is the solution set for the statement: $x < 1$ and $x + 5 \leq 2x + 8$ if only whole numbers are considered?

   a. $\{-3, -2, -1, 0, 1\}$

   b. $\{-3, -2, -1\}$

   c. $\{-3, -2, -1, 0\}$

   d. $\{-2, -1, 0, 1\}$

   e. none of these

2. If a, b and c are integers, which of the following is false?

a. If $a < b$ and $b < c$ then $a < c$.

b. If $c < 0$ and $a < b$ then $ac < bc$.

c. If $a < b$ then $a + c < b + c$.

d. If $c > 0$ and $a < b$ then $ac < bc$.

e. None of these

3. If $a < b$, which of the following is true for all a's and b's?

a. $\frac{a}{8} > \frac{b}{8}$

b. $\frac{8}{a} < \frac{8}{b}$

c. $a^2 < b^2$

d. $a < \frac{a + b}{2} < b$

e. None of these

4. $|x|$ means the "absolute value of x."

$|x| = x$ if $x \geq 0$ and $|x| = -x$ if $x < 0$. What values may be used to replace x in the statement $|x - 3| = 1$?

a. +4, +2

b. 3, -3

c. -2, -4

d. 4, -3

e. None of these

5. What inequality may be used to replace the statement $5 \geq x \geq 0$ and $3 < x < 7$?

a. $3 < x < 7$

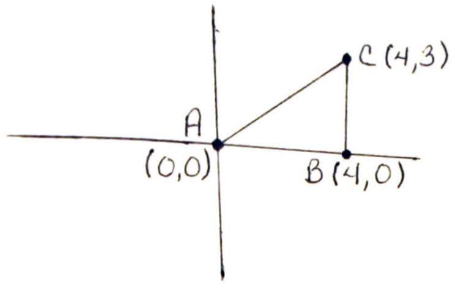
b. $3 \leq x \leq 5$

c. $0 \leq x < 7$

d. $3 < x \leq 5$

e. None of these

6.



a. 5

b. $\sqrt{5}$

c. $\sqrt{24}$

d. $\sqrt{12}$

e. None of these

What is the distance from A to C?

7. The graph of the equation $(x + 2)^2 + (y - 5)^2 = 25$ is

a. an ellipse

b. a parabola

c. a hyperbola

d. a circle

e. None of these

8. If $f(x) = 2x + 3$, $f(y)$ is

a. $\frac{1}{2}(y - 3)$

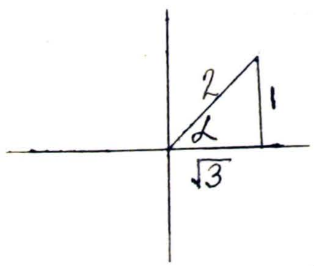
b. $2y + 5$

c. $3 - 2x$

d. $x - 3y$

e. None of these

9.



What is the sine of α in the above triangle?

a. $\frac{1}{2}$

b. $\sqrt{3}/2$

c. $\sqrt{3}/3$

d. 2

e. None of these

10. The cosine of θ is zero when θ is

a. $\pi/2$

b. 0

c. $3\pi/4$

d. $\pi/3$

e. None of these

11. If $y = x^n$ then $y' = nx^{n-1}$. If $y = x^3$ then y' is

a. $3x^3$

b. $2x^3$

c. $3x^2$

d. $2x^2$

e. None of these

12. If $y = x^3 + 2x^2 - 3x + 4$, what is y when x is 2?

a. 36

b. 10

c. 14

d. 4

e. None of these

13. The slope of the line $y = 2x + 3$ is

a. $\frac{1}{2}$

b. $\frac{2}{3}$

c. 3/2

d. 3

e. None of these

14. If $v = \frac{\pi r^2 h}{3}$ and $r = \frac{1}{4}h$, then v in terms of h is

a. $(4\pi r^3)/3$

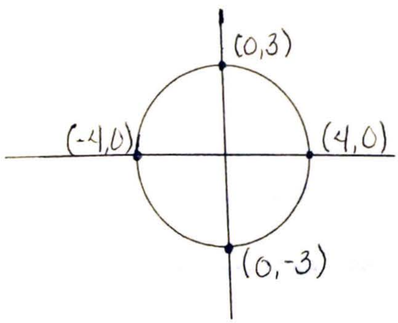
b. $(\pi h^3)/192$

c. $(3\pi r^3)/4$

d. $(\pi h^3/48$

e. None of these

15. The equation of the following ellipse is



a. $9x^2 + 16y^2 = 1$

b. $\frac{x^2}{9} + \frac{y^2}{16} = 1$

c. $\frac{x^2}{16} + \frac{y^2}{9} = 1$

d. $3x + 4y = 12$

e. None of these

16. If $f(x) - 2x + 3$, $\frac{f(x+h) - f(x)}{h}$ is

a. $\frac{f(h)}{h}$

b. $2h + 3$

c. 1

d. 0

e. None of these

17. The coordinates of a point <u>not</u> on $x^2 - 8x + y^2 - 6y = 0$ are

a. (0,0)

b. (4,8)

c. (8,0)

d. (4,-2)

e. None of these

18. The equation for the amount of work done in stretching the free end of a spring from point <u>a</u> to point <u>b</u> is $W = \frac{k}{2}(b^2 - a^2)$ where k is the spring constant. If $a = 2$ in., $b = 3$ in., and $w = 9$ inch pounds, k is

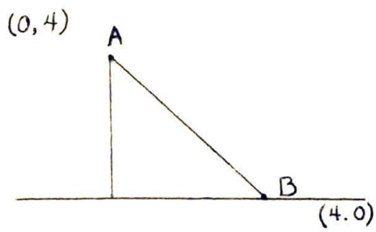
a. $4\frac{1}{2}$ pounds/inch

b. 18 pounds/inch

c. 45 pounds/inch

d. 18/5 pounds/inch

e. None of these

19. 

The equation of line AB is

a. $x^2 + y^2 = 16$

b. $x + y = 4$

c. $y - x = 4$

d. $x^2 + 2xy + y^2 = 16$

e. None of these

20. The point of intersection of $x - y = 8$ and $2x + 3y = 1$ is

a. (-3, 5)

b. (9, 1)

c. (1, -7)

d. (5, -3)

e. None of these

21. For what value of x is the equation $y = 3x^2$ a minimum?

a. $\frac{1}{3}$

b. 1

c. 2

d. 0

e. None of these

22. The product of $2 + 3\sqrt{2}$ and $3 + 4\sqrt{2}$ is

a. $6 + 9\sqrt{2}$

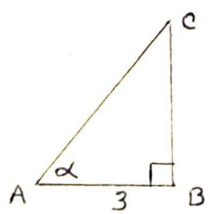
b. $6 + 29\sqrt{2}$

c. $30 + 9\sqrt{2}$

d. $6 + 17\sqrt{2}$

e. None of these

23.



The sine of α is .600. What is the length of line segment $\overline{CB}$?

a. 5.0

b. 4.0

c. 1.8

d. 3.0

e. None of these

24. $(x - y)(x + y)(x^2 + y^2)(x^4 + y^4)$ equals

a. $x^8 + 16x^7y^1 - 32x^3y^3 + 16xy^7 - y^8$

b. $x^8 + y^8$

c. $x^8 - y^8$

d. $x^8 - 2xy + y^8$

e. None of these