AN EXAMINATION OF MIDDLE SCHOOL STUDENTS' SENSE MAKING STRATEGIES

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I am submitting herewith a field study written by Laura Garza Atkins entitled "An Examination of Middle School Students' Sense Making Strategies." I have examined the final copy of this field study for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Education Specialist, with a major in Education.

Mary Lou Witherspoon

Dr. Mary Lou Witherspoon,
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AN EXAMINATION OF MIDDLE SCHOOL STUDENTS' SENSE MAKING STRATEGIES

A Field Study
Presented to the
Graduate and Research Council of
Austin Peay State University

In Partial Fulfillment
of the Requirements for the Degree
Education Specialist

by
Laura Garza Atkins
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ABSTRACT

A study was conducted to examine the sense-making strategies of middle school students. Sixth grade students responded to pre-test and post-test questionnaires. The data were collected and categorized using qualitative methods.

A review of the professional literature indicates that many students demonstrate a lack of number sense. Students that have been trained in procedure-oriented classrooms do not attempt to examine their solutions for reasonableness. Students abandon much of the informal knowledge they possess in order to survive in classrooms in which sense making is not valued. Thus many students are simply unable to develop concepts on their own after algorithms and procedures have been taught.

Interviews with teachers were conducted to determine teacher beliefs and expectations. Interviews with teachers revealed that a great majority of the teachers involved did not value number sense and/or sense-making. Most of the teachers felt that they simply did not have time to help students make sense of mathematics.

Most of the students used absolutely no sense-making strategies either before or after the instructional unit designed to promote sense-making. Interviews with the teachers revealed that the teachers involved in the study lacked sufficient knowledge of basic mathematical concepts and related mathematics teaching strategies. The only method of instruction regularly used by the teachers involved can be characterized as a "drill and practice" model.

Concentration on the state mandated assessment appears to drive the mathematics instruction of these teachers instead of the thinking and learning of students. Teachers said they do not feel secure using different methods even though the methods currently being employed are not working for most of the students. Most of these teachers have no plans for changing the manner in which they teach mathematics.

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CHAPTER 1

Introduction

Background

Recently there has been increased attention by mathematics educators over the importance of developing sense-making strategies and number sense in students (Markovits and Sowder, 1994; Silver et al, 1993; Greeno, 1991). These studies reveal that many students demonstrate a lack of number sense, have an inadequate understanding of basic number concepts, and make little connection between mathematics and the real world. The National Council of Teachers of Mathematics (NCTM) states that number sense refers to "an intuitive feeling for numbers and their various uses and interpretations; an appreciation for various levels of accuracy when figuring; the ability to detect arithmetical errors; and a common-sense approach to using numbers" (NCTM, 1991, p. 3).

Number sense is a well organized conceptual network that enables one to relate number problems in flexible and creative ways (Sowder, 1992). Number sense is not something that students either have or do not have but something that grows as students are engaged in mathematical activity. It is also clear that number sense does not develop accidentally. Number sense "evolves from concrete experiences and takes shape in oral, written, and symbolic expression" (National Research Council, 1989, p. 84). Many students who successfully manipulate symbols using standard algorithms may not demonstrate number sense in problems solving situations. This deficiency inhibits students' ability to make decisions concerning the relationships among numbers. It also affects the ability of students to give valid reasons for their decisions (Leutzinger and Bertheau, 1989) and to detect computational errors through interpretation of their solutions (Silver et al, 1993).

When the focus of mathematics instruction is memorization of basic facts and mastery of computational procedures, students are not encouraged to explore and investigate the relationships among numbers. The standard computational algorithms of elementary school mathematics were developed because people needed fast, efficient computational schemes. Since inexpensive calculators can now do most of the arithmetic needed for daily life, a world of opportunities opens up for the elementary school mathematics curriculum (Markovits and Sowder, 1994). Today's technology driven society requires that everyone have a good grasp of chance, of reasoning, of estimating, and of patterns (National Research Council, 1989). These important "new basics" should now be included in the elementary school mathematics classes.

Most students view mathematics as a set of rules and procedures in which problems are solved by applying the appropriate computational algorithms which have been taught by the teacher. Students expect these algorithms to be routine tasks which, if performed correctly, will always yield the correct answer. However, many students who are algorithmically trained and oriented have no concept of the reasonableness of their answers (Markovits & Sowder, 1994). Most students who become accustomed to coping with mathematics from a procedural perspective are uncomfortable when they encounter situations where there is more than one appropriate answer and more than one appropriate method for solving a problem. When asked to estimate, justify, conjecture, think, and explain, many students reach an impasse and are unable to even make an attempt at the problem. Many other students simply apply inappropriate algorithms (Markovits, 1987).

Number sense will be valued among students only if educators believe that it is more important for students to make sense of the mathematics than to master mindlessly rules and algorithms. Mathematics instruction should be aimed at conceptual understanding and at developing in students the ability to apply their mathematical knowledge with "flexibility and resourcefulness" (Schoenfeld, 1992). When instruction and assessment focus on developing number sense, students gain mathematical power and the belief that mathematics should and can make sense (Markovits and Sowder, 1994).

Research Questions

The following research questions were investigated:

1. Before an instructional unit to promote sense-making is implemented, what strategies, if any, do students use?

1. Before an instructional unit to promote sense-making is implemented, what strategies, if any, do students use?

3. What are teachers' perceptions about how well students will perform on the pre-test?

4. What teacher suggestions, including assessment issues, are offered in the development of the instructional unit?

5. What impact do teachers predict the instructional unit will have on students?

6. What impact can the instructional unit have on students?

All of the research questions were answered through interviews with teachers and through the examination of student responses to the pre-test and post-test questionnaires.

CHAPTER 2

Review of Literature

Rationale for School Mathematics

One of the major rationales for the teaching of mathematics has traditionally been the notion of mental discipline. Dating back to Plato, the theory was that those who are good at mathematics are usually good thinkers; therefore, those who are schooled in mathematics will be good thinkers. As exercise and discipline train the body, mental exercise and discipline train the mind. The result is better thinkers (Schoenfeld, 1992).

Elementary school became the place where children learn the mathematical skills needed for daily life. Earlier this century, arithmetic was an adequate objective for elementary school mathematics since, for most people, mathematics in daily life required little more that arithmetic. While the goal of elementary education has not changed, the mathematical objectives appropriate to this goal are very different now from what they were half a century ago (National Research Council, 1989).

Definition of Number Sense

According to the National Research Council (1989, p. 4) "the major objective of today's elementary school mathematics should be to develop number sense." Number sense is considered vital in elementary school mathematics (NCTM, 1989, p. 95) and yet its exact meaning is somewhat elusive. "Number sense" seems to be an umbrella term that refers to an insightful, reflective approach to doing arithmetic (Schoen, 1989). Number sense may be best described by looking at the characteristics of those who demonstrate number sense. Schoen (1989) stated that

> students are said to have good number sense if they routinely apply, in appropriate situations the concept of number, the relationship among and between numbers, the properties of numbers under the various operations, the effect on numbers of each operation, and the role of numbers as measures of various quantities in real-world settings (p. 67).

Number sense involves subtle judgment and interpretation, the application of multiple criteria, uncertainty, self-regulation of the thinking process, and imposing meaning (Resnick, 1989).

Resnick (1989) describes number sense as "nonalgorithmic, complex, and effortful." Carpenter (1989) argues that a critical aspect of number sense is the ability to operate with numbers flexibly. Number sense, like common sense, requires multi-dimensional input (Marshall, 1989). Number sense is about making sense of numbers and things mathematical (Silver, 1989). Number sense includes flexible mental computation, numerical estimation, and quantitative judgment. The capabilities that are associated with number sense go beyond knowing facts and procedures (Greeno, 1991).

Number sense is not static but grows gradually. Some growth of number sense takes place in all cultures; thus, number sense is both natural and universal (Case, 1989). Number sense is "both the ability of the learner to make logical connections between new information and previously acquired knowledge and the drive within the learner to make forming these connections a priority" (NCTM, 1991, p. 5). A student with good number sense has an automatic monitoring system for the reasonableness of his/her arithmetic results (Schoen, 1989). A student with number sense is expected to have in mind that there is not always just one answer and there is not always just one algorithm (Markovits, 1989).

Reasonable Answers

The National Council of Supervisors of Mathematics (1977) mentioned the importance of attention to reasonable answers. Teachers should encourage the use of estimating skills to assess what a reasonable answer might be (NCSM, 1977). In estimation there is usually no such thing as the correct response. There are reasonable responses and unreasonable responses. A reasonable response in one situation might be unreasonable in an apparently similar situation. There are situations for which no algorithm exists, but instead reasonable judgment is required to solve the problem. Checking the reasonableness of the answer obtained is an important component of estimating. In a sense, estimation requires more flexible thinking than other school mathematics topics because students must consider many different answers as correct (Markovits, 1987).

Almost never practiced is the idea that the last phase in the performance of a mathematical task is the validation of the solution. In order to solve the estimation tasks there is a need for reasonable judgment and common sense. Checking the reasonableness of an answer is usually left undone and students might give nonsensical answers. Often when students check their answers they are simply running through the steps of the algorithm and not checking for reasonableness. In similar problems with different contexts, many students use algorithms automatically, without any thought to the context and do not seem to notice unreasonable answers. Checking for reasonableness almost always requires no computation but instead, reasonable judgment and common sense are essential. (Gagne, 1983).

Nonsense versus Number Sense

Few students exhibit number sense when solving arithmetic problems in school (Markovits & Sowder, 1994). Most students possess number sense with whole numbers, but, when they encounter symbolic and rule-driven mathematics, students ignore their informal insights and attempt to deal with number tasks within the more formal mathematics arena (Trafton, 1989). Too often children learn mathematics as standard algorithms which are often not well understood at the time they are learned. Procedures are nonsensical from the student's point of view are likely to minimize the development of number sense (Case, 1989; Reys, 1989). Students are unable to link adequately formal rules to mathematical concepts that give symbols meaning and link mathematics to real-world situations. Students who are mechanically trained have no concept of the reasonableness of their answers. This inability is not a reflection of their capacity to think logically but instead the result of earlier training (Sowder & Kelin, 1993).

Many students believe that mathematics is not something that they can make sense of but rather something almost completely arbitrary. In order to cope with mathematics, students have learned to memorize without looking for meaning because the meaningfulness of mathematics is made inaccessible to most of them (Lampert, 1990). Even when standard algorithms are understood they are usually executed in a mindless fashion (Case, 1989).

For many, mathematical knowledge is seen as a body of facts and procedures and knowing mathematics is seen as having mastered all of these facts and procedures (Schoen, 1989). Lampert (1990) asserts that too often "doing mathematics means following the rules laid down by the teacher; knowing mathematics means remembering and applying the correct rule; and mathematical truth is determined when the answer is ratified by the teacher." Beliefs about how to do mathematics and what it means to know mathematics are acquired through years of watching, listening, and practicing in school (Lampert, 1990). Schoen (1989) contends that an elementary mathematics curriculum that focuses primarily on the procedures and algorithms of arithmetic "trivialize mathematics" and is "severely impoverished" (Schoen, 1989).

When analyzing the mediocre performance of U.S. mathematics students on the recently administered Third International Mathematics and Science Study (National Center for Educational Statistics, 1996, p. 121), the data show that fifty percent of the U.S. students agree or strongly agree that doing well in mathematics depends on "natural talent or ability." Thirty-two percent of the U.S. students agree or strongly agree that doing well in mathematics depends on "good luck" and fifty-nine percent agree or strongly agree that doing well in mathematics depends on "memorizing the textbook or notes." Only fifty-seven percent of seventh graders and sixty-six percent of eighth graders responded correctly to a rounding problem involving fractions and numbers sense (International Association for the Evaluation of Educational Achievement, 1996, p. 59). As reported in *Pursuing Excellence: A Study of U.S. Eighth- Grade Mathematics and Science Teaching, Learning, Curriculum, and Achievement in International Context* (National Center for Educational Statistics, 1996, p. 42) sixty percent of U.S. teachers listed steps typical of their eighth-grade mathematics lessons as "(1) the teacher instructs the students in a concept or skill, (2) the teacher solves problems with the class, and (3) the students practice on their own with the teacher assisting individual students."

Unfortunately, many U.S. educators have apparently failed to be convinced of the importance of fostering the development of number sense in the elementary and middle school mathematics classrooms. The 1996 National Assessment of Educational Progress (National Center for Educational Statistics, 1997, P. 4) demonstrated the increased importance given to developing number sense in that approximately half the questions at each grade level involved some aspect of estimation.

Mathematics has been characterized as the "science of patterns" (Schoen, 1989). Although the "language of mathematics" is based on conventions that must be learned, students must move beyond rules in order to express themselves using the language of mathematics. This shift suggests drastic changes in both curricular content and instructional methods. School mathematics should involve "seeking solutions, not just memorizing procedures; exploring patterns, not just memorizing formulas; formulating conjectures; not just doing exercises" (National Research Council, 1989, p. 84).

As articulated in *Everybody Counts* (National Research Council, 1989, p. 84) "Number sense builds on arithmetic as words build on the alphabet. Numbers arise in measurement, in chance, in data, and in geometry, as well as in arithmetic. Mathematics in elementary school should weave all these threads together to create in children a robust sense of numbers." Classrooms must become communities in which mathematical sense making is practiced. Schoenfeld (1992) insists that teachers must create a classroom atmosphere in which all students feel comfortable to experiment and are invited to explain their thinking.

In *Developing Number Sense* (NCTM, 1991) an emphasis is placed on the classroom environment, "In seeking to develop number sense in their students, teachers must create a classroom environment that encourages student exploration, questioning, verification, and sense-making. In classrooms where number sense is a priority, students are active participants who share their hypotheses, reasoning, and conclusions and teachers become guides and moderators instead of dispensers of answers. The emphasis shifts from finding the specific solution to investigating how the answer was obtained" (NCTM, 1991, p. 5).

Research on Number Sense

In recent years, increased research interest in number sense has been seen. Markovits and Sowder (1994) reported that written measures and interviews before instruction, immediately after instruction, and several months later, revealed that after instruction students were more likely to elect to use strategies that reflected number sense and that this was a long-term change. The amount of change in behavior that occurred after essentially limited instruction was believed to be the result of not a great deal of learning of radically new concepts but rather, existing knowledge began to be used in new ways. Intuitive notions of number were called to the surface and new connections were formed. It appeared to the investigators that the students reorganized and used existing knowledge rather that acquiring new knowledge structures.

Silver, Shapriro, and Deutsch (1993) observed that student performance was adversely affected by students' disassociation of sense making from the solution of school mathematics problems and students' difficulty in providing written accounts of their mathematical thinking and reasoning.

Mack (1990) noticed that knowledge of rote procedures frequently interfered with students' attempts to build on their informal knowledge. The results of the study suggested that knowledge of rote procedures interferes with students' attempts to construct meaningful algorithms. Students in the study focused on symbolic manipulations, whether correct or incorrect, in situations where they possessed knowledge of rote procedures. The results, however, did not suggest that the influence of rote procedures cannot be overcome, but a great deal of time and directed effort is needed to encourage students to draw on informal knowledge rather than use rote procedures. The results seem to suggest that teaching concepts prior to procedures may be beneficial and that students can construct meaningful algorithms by building upon informal knowledge.

Markovits (1987) reported that after a short treatment, students demonstrated improvement in absolute and relative error and reasonable answers. After the treatment many students performed better than the preservice teachers involved in the research project. For topics which involve familiarity with everyday situations, the influence of the treatment was significant but small. Students involved in the research project have difficulty accepting the possibility that there may be more than one answer and one kind of algorithm. Other student difficulties mentioned were decision-making, connecting mathematics with reality, and inattention to very unreasonable answers.

Fennema et al (1993) observed that estimation instruction did not negatively effect the computation skills or the number of word problems students could solve correctly. There was evidence that students were better able to estimate the solutions of word problems after the estimation instruction.

CHAPTER 3

Methodology

Subjects

The subjects for this study included sixth grade students and their mathematics teachers in a public Tennessee school. The selected public school is attended by approximately 950 students, about half of these students are sixth graders. All of the middle school students in the county-wide system except approximately 100 students attend the selected school. Less than five percent of the student population is African-American.

All sixth grade students in the selected school participated in the study; however, the data from only thirty students were analyzed. Within the structure of the school, students are tracked into three levels of mathematics classes. The three levels are referred to as "low," "average," and "high." Two sets of fifteen students from two randomly selected teachers were analyzed. One set of students was selected from a "high" track and one set of students was selected from a "low" track. All of the sixth grade teachers participated in the interviews.

Procedure

Student responses to pre-test and post-test questionnaires were collected and categorized using qualitative methods. A constant comparison method was used to categorize the justification for solutions students gave (Lincoln & Guba, 1985). The items in the pre-test questionnaire were used in a previous, unpublished study conducted by Markovits (1987). However, the items were modified to encourage more written explanations from students.

The items in the post-test questionnaire were based upon the same unpublished study.

Each student was assigned a number to identify his/her work and to maintain student anonymity. System, school, teacher, and student anonymity has been maintained in this report.

The pre-test questionnaire was given to students before the teachers participated in professional development activities. Teachers were asked not to help students with responses on the pre-test questionnaire.

Before the post-test questionnaire was given, teachers were involved in four discussions which were approximately one-hour in length. These discussions were related to students' number sense development and sense-making strategies. Activities to help students make sense of mathematics were demonstrated. Teachers explored the concepts of decimal arithmetic using grid paper, base-ten blocks, and calculators. During the sessions, much of the time was spent responding to teacher questions about operations on fractions and decimals. Teachers explored fraction concepts using fraction circles, pattern blocks, and tangrams. These and other activities were designed to help teachers understand the concepts behind procedures they were currently teaching. Teachers were required to view, during their planning period, the video series created by NCTM called, *Number Sense, Now* (NCTM, 1993).

Teachers formed questions that they planned to ask students to encourage students to examine their answers for reasonableness. Teachers were encouraged to involve students as much as possible in the discussions of the pre-test questionnaire. Before the post-test questionnaire was given, teachers had approximately six class meetings to draw students' attention to reasonable answers and sense making strategies as a part of the regular classroom instruction. During the administration of the post-test questionnaire, teachers were asked not to help students with responses. Not until the end of the project did teachers know whether their students were going to be selected for analysis.

Limitations

The extent to which individual teachers changed their instruction was optional and not observed by the researcher. Teachers that did not value the development of number sense and sense making strategies in students may have affected the motivation level of the students involved. Low expectations on the part of teachers may have been communicated to the students.

Many student responses did not offer insight to procedures or algorithms used to obtain both appropriate and inappropriate answers. Had students been available for interviews, more information might have been revealed about students' thinking.

CHAPTER 4

Results

Comparisons: Pre- and Post-Test Question 1

Pre-test: 5.2 X 3.2 = ?

a) .0176 b) .176 c) 1.76 d) 17.6 e) 176

Post-test: 24.196 x 0.955 = ?

a) 0.2310718 b) 2.310718 c) 23.10718 d) 231.0718 e) 2310.718

Although students were instructed not to use the multiplication algorithm, many did. A close inspection indicates that the "low" group was more persistent in using the multiplication algorithm. A majority of the responses of the "high" group indicate that they followed directions and did not multiply. More students in the "high" group actually chose the incorrect answer than did the "low" group. However, most of the "high" group simply counted the number of digits behind the decimal points, which in this case, produced incorrect answers. On the post-test there was no significant change in the responses of the "high" group. The "low" group seemed to resort to counting digits behind the decimal point. More of the "low" group missed post-test question 1 than missed pre-test question 1.

As shown in Figure 1, fourteen of thirty students used the multiplication algorithm in the margins of the pre-test. Twelve students showed evidence that they counted the number of digits behind the decimal and used that number to determine the placement of the decimal in the product. It was unclear in the students' justification what method was used by the other four of the students.

On the post-test, none of the students showed evidence of using the multiplication algorithm. Twenty students showed evidence that they counted the number of digits behind the decimal and used that number to determine the placement of the decimal in the product. It was unclear in the students' justification what method was used by the other ten of the students.

Figure 1: Comparison of the "High" & "Low" group responses

	Question 1	Pre-test	
Choose one answer and explain why you chose it. 5.5 x 3.2 =			
	correct answer selected	incorrect answer selected	other
	used multiplication algorithm	counted decimal places	
Low/Pre-test (n = 15)	11	0	4
High/Pre-test (n = 15)	3	12	0
	Question 1	Post-test	
Without multiplying choose the best answer. 24.196 x 0.955 =			
Low/Post-test (n = 15)	0	7	8
High/Post-test (n = 15)	0	13	2

<u>Comparisons: Pre- and Post-Test Question 2</u>

Pre-test: 426.5÷ 0.469 is

a) less that 426.5 b) equal to 426.5 c) greater that 426.5

Post-test: 600 ÷ .521 is

a) equal to 600 b) greater than 600 c) less than 600

The most common response to this question was "less than" on both the pre- and post-test. However, more students in the "low" group moved to the most popular incorrect response on the post-test. Many of the "low" group simply left this question unanswered and unattempted.

Several of the "high" group actually began moving the decimal using the division algorithm. These students were then able to determine that the quotient was greater than 426.5. Most of the students in both groups who selected "less than" gave the following reason: "when you divide it always makes the number smaller."

On the pre-test, as shown in Figure 2, fifteen of thirty students indicated in their justifications that the division of a number always yields a quotient that is less that the dividend. The remainder of the students either did not indicate what method or strategy they used or the description of the method used was unclear.

On the post-test, nineteen of thirty students indicated in their justifications that the division of a number always yields a quotient that is less that the dividend. The remainder of the students either did not indicate what method or strategy they used or the description of the method used was unclear.

Figure 2: Comparison of the "High" & "Low" group responses

	Question 2 / Pretest	
Choose one answer and explain why you chose it. $426.5 \div 0.469 =$		
	incorrect answer selected	other
	less than	
Low/Pre-test (n=15)	6	9
High/Pre-test (n=15)	9	6
	Question 2 / Post-test	
Without dividing, choose the best answer and explain why you chose it. $600 \div .521 =$		
Low/Post-test (n=15)	11	4
High/Post-test (n=15)	8	7

Comparisons: Pre- and Post-Test Question 3

Pre-test: The height of a 10-year-old boy is 1.5 meters. What do you think his height will be when he is twenty?
Post-test: The shoe size of a 5-year-old boy is 9. What do you think his shoe size will be at age 10?

The "low" group on the pre-test used several different approaches to attempt to solve the height problem including:

$$10 \times 1.5 = 15$$

$$1.5 \times 1.5 = 2.25$$

Most of the "high" group, however, applied the multiplication algorithm using $1.5 \times 2 = 3.0$. On the post-test the "low" group again use a variety of inappropriate strategies. Most of the "high" group incorrectly chose to multiply $9 \times 2 = 18$.

It is interesting to note that almost as many of the "high" group missed this question as the "low" group in both the pre- and post-tests.

On the pre-test, as shown in Figure 3, fourteen of thirty students indicated in their justifications that they used the multiplication algorithm to arrive at their answer. Sixteen of thirty students either did not indicate what method or strategy they used or the description of the method used was unclear.

On the post-test, thirteen of thirty students indicated in their justifications that they used the multiplication algorithm to arrive at their answer. On the post-test, seventeen of thirty students either did not indicate what method or strategy they used or the description of the method used was unclear.

Figure 3: Comparison of the "High" & "Low" group responses

	Question 3 / Pre-test	
The height of a 10-year-old boy is 1.5 meters.		
What do you think his shoe size will be at age 10? Explain your answer.		
	incorrect answer /1.5 x 2 = 3.0	other
Low/Pre-test (n=15)	4	11
High/Pre-test (n=15)	10	5
	Question 3 / Post-test	
The shoe size of a 5-year-old boy is 9.		
What do you think his shoe size will be at age 10? Explain your answer.		
	incorrect answer/ 9 x 2 = 18	
Low/Post-test (n=15)	4	11
High/Post-test (n=15)	9	6

Comparisons: Pre- and Post-Test Question 4

Pretest: A book store delivered 188 books to 6 libraries. How many books do you think each library received?

Post-test: A beverage company produces 165 cans of soda. How many 6 packs of soda can be filled with the 165 cans of soda?

The "high" group out-performed the "low" group on pre-test question 4. However, only one of the "high" group discussed the remainder. There were ten correct responses in the former and seven in the latter. The same number of students in the "high" group missed post-test question 4. The only consistency between both groups was that the responses indicated problems with the division algorithm. Seven from the "high" group and four from the "low" group correctly answered post-test question 4.

On the pre-test, as shown in Figure 4, four of thirty students selected the correct answer but did not discuss the remainders in their justifications. Twenty-six students on the used either the division algorithm or the description of the method used was unclear.

On the post-test only one of thirty students selected the correct answer but did not discuss the remainder in his/her justification. Twenty-nine students on the post-test used either the division algorithm or the description of the method used was unclear.

Figure 4: Comparison of the "High" & "Low" group responses

	Question 4 / Pre-test	
	A bookstore delivered 188 books to 6 libraries.	
How many books do you think each library received? Explain your answer.		
	Correct Answer / Did not discuss the remainder	Other
Low/Pre-test (n=15)	1	14
High/Pre-test (n=15)	3	12
	Question 4 / Post-test	
	A beverage company produces 165 cans of soda.	
How many 6 packs of soda can be filled with the 165 cans of soda? Explain your answer.		
Low/Post-test (n=15)	1	14
High/Post-test (n=15)	0	15

Comparisons: Pre- and Post-Test Question 5

Pre-test: A bath has two outlets. The first alone empties the bath in 10 minutes, and the second alone in 4 minutes. Both were opened at the same time. In how many minutes will the bath be empty?

a) about 14 min b) about 40 min c) about 6 min d) about 4 min e) about 3 min

Post-test: Susan can mow the yard in 45 minutes. Joe can mow the same yard in 90 minutes. Mowing together, how long would it take Susan and Joe to mow the same yard?

a) about 4050 min b) about 45 min c) about 30 min d) about 90 min e) about 135 min

There was only one correct response from the "low" group and three from the "high" group. On the pre-test the two most frequent incorrect answers of the "low" group were given as a result of choosing the wrong operation. Some of the "low" group attempted to solve the problem by using addition while others of the "low" group attempted to solve the problem by using subtraction. The "high" group experienced just as much difficulty with this question as the low group. Most of the "high" group attempted to solve the problem on both the pre- and post-test using addition. On the post-test five "low" group students answered correctly as did eight "high" group students.

On the pre-test, as shown in Figure 5, fourteen of thirty students selected the incorrect answer. These fourteen students used the subtraction algorithm in the margins of the pre-test. Ten of thirty students students selected the incorrect answer. These ten students used the addition algorithm in the margins of the pre-test. The description of the method used of the remaining ten students on the pre-test was unclear.

On the post-test, as shown in Figure 5, ten of thirty students selected the incorrect answer. These ten students used the subtraction algorithm in the margins of the pre-test. Ten of thirty students students selected the incorrect answer. These ten students used the addition algorithm in the margins of the pre-test. The description of the method used of the remaining ten students on the post-test was unclear.

Figure 5: Comparison of the "High" & "Low" group responses

	Question 5	Pre-test	
A both has two outlets. The first alone empties the bath in 10 minutes, and the second alone in 4 minutes			
Both were opened at the same time. In how many minutes will the bath be empty? Choose the best answer. Explain			
	used subtraction	used addition	other
Low/Pre-test (n=15)	5	7	3
High/Pre-test (n=15)	9	3	3
	Question 5	Post-test	
Susan can mow the yard in 45 minutes. Joe can mow the same yard in 90 minutes.			
Mowing together, how long would it take Susan and Joe to mow the same yard? Chose the best answer. Explain			
Low/Post-test (n=15)	1	7	7
High/Post-test (n=15)	9	3	3

Teacher Interviews

Interviews with teachers revealed that a majority of the teachers involved with the project did not value number sense and/or sense-making. Most of the teachers felt that they simply did not have time to help students make sense of mathematics. The overriding concern of all of the teachers was preparing students for the state-mandated Tennessee Comprehensive Assessment Program (TCAP). The teachers were not convinced that sense-making strategies could make a difference in their students' performance in class as well as on the TCAP.

The teachers had many of the same difficulties with the pre- and post-tests as the students did. For example, most of the teachers believed that division will always result in a smaller quantity. After some explanation most of the teachers seemed to understand the purpose of the questions selected and what answers were appropriate.

Research Questions

Before an instructional unit to promote sense-making is implemented, what strategies, if any, do students use?

Most of the students in both the "high" and "low" groups used absolutely no sense-making strategies before the instructional unit designed to promote sense-making.

After an instructional unit to promote sense-making is implemented, what strategies, if any, do students use?

There is no evidence about the degree to which teachers implemented the information they received. Given the teachers' perceptions about number sense and its value, it is certainly possible some chose not to implement the suggested activities. In fact there is evidence that the teachers were not receptive to the information because of their perceptions of the importance of sense-making, the pressure of TCAP, and their level of understanding. The findings suggest that whatever strategies and methods teachers chose to implement in their classrooms were ineffective.

Only two or three of the thirty students, all from the "high" group, seemed to benefit from the instructional unit designed to promote sense-making. Some students from the low group actually seemed to perform worse after the instructional unit.

What are teachers' perceptions about how well students will perform on the pre-test?

Teachers of the "low" group articulated low expectations for their students. Teachers of both groups felt that the questions were unfair because questions of this type had never been included in their lessons and/or textbooks and certainly not on the TCAP. Teachers communicated a concern that problems similar to those on the pre- and post-tests would not help their students do better on the TCAP. Teachers also reported that their students were very uncomfortable with the pre- and post-test questions.

What teacher suggestions, including assessment issues, are offered in the development of the instructional unit?

None of the teachers had appropriate suggestions for the instructional unit designed to promote sense-making. The unit did seem to benefit some of the teachers. Some communicated that they understood topics they had never understood before.

What impact do teachers predict the instructional unit will have on students?

Teachers of the "high" students believed that their students would make a significant improvement. These teachers seemed baffled when the results of the post-test were reviewed. Teachers of the "low" students believed that nothing could help their students because the "low" students were so far behind the "high" group.

What impact can the instructional unit have on students?

The instructional unit did not seem to have an impact on the "high" students. The "low" students, however, seemed to be affected by the instructional unit. After the unit, the "low" students used many of the same inappropriate algorithms that the "high" students used. Without a consistent emphasis by the teacher on sense-making , students seem unable or unwilling to apply such strategies in mathematics classes.

CHAPTER 5

Summary, Discussion, and Recommendations

Summary

This study was conducted to examine the sense-making strategies of middle school students. In order to accomplish this, sixth grade students responded to pre-test and post-test questionnaires. The data were collected and categorized using qualitative methods. Most of the students used absolutely no sense-making strategies either before or after the instructional unit designed to promote sense-making.

Interviews with teachers were conducted to determine teacher beliefs and expectations. Interviews with teachers revealed that a great majority of the teachers involved did not value number sense and/or sense-making. Most of the teachers felt that they simply did not have time to help students make sense of mathematics.

Discussion

The analyses within this study were for two main purposes. First, the researcher sought to determine what sense-making strategies, if any, students use before and after instruction aimed at developing sense-making. Secondly, the researcher sought to determine teachers' perceptions about their students. However, the study evolved into the measure of typical professional development strategies demonstrated to teachers. Teacher beliefs and perceptions about number sense, the level of content understanding by teachers, and the pressures of state assessment on teachers proved to be obstacles to the study.

A review of the professional literature indicates that many students demonstrate a lack of number sense. Students who have been trained in procedure-oriented classrooms do not attempt to examine their solutions for reasonableness. The mindless execution of algorithms and procedures on the part of many students actually is their only approach for coping with mathematics. Students abandon much of the informal knowledge they possess in order to survive in classrooms in which sense making is not valued. Thus many students are simply unable to develop concepts on their own after algorithms and procedures have been taught.

Examination of the data collected from students in this study, unfortunately, did not produce in any unexpected responses from students on the pre-test. However, the results of the post-test indicated that the short instructional unit designed to promote sense-making had no impact on most students and might have had an adverse effect on some of the students. There were a few observed differences between the "high" and "low" group performance. The "high" group responded inappropriately to about the same number of questions as did the "low" group. Students in the "high" group were better able to communicate their strategies; however, they seemed more reliant upon rules and procedures in their explanations than did the "low" group. Students of the "high" group left fewer questions unanswered than did the "low" group. Many of the responses of the "low" group could not be categorized because of the lack of explanations.

Interviews with the teachers revealed that the teachers involved in the study lacked sufficient knowledge of basic mathematical concepts and related mathematics teaching strategies. The only method of instruction regularly used by the teachers involved can be characterized as a "drill and practice" model. Concentration on the state mandated assessment appears to drive the mathematics instruction of these teachers instead of the thinking and learning of students. These teachers' excuse for not implementing sense-making strategies in the classroom was the TCAP. Some of the teachers said they do not feel secure using different methods even though the methods currently being employed are not working for most of the students. Most of these teachers have no plans for changing the manner in which they teach mathematics. This out-dated method of teaching mathematics is the same method by which most of them "learned" mathematics and it is the only method they feel comfortable using in their classrooms.

Perhaps the most important finding of this study was that professional deveopment that presents teaching strategies for a particular objective cannot be effective if teachers' beliefs about the objective are counterproduct and if teachers do not have the requisite content understanding.

High quality middle grades mathematics instruction is vital to the future academic success of all students. Assignment of middle grades teachers to mathematics positions should be made only after serious consideration of the teacher's background, experience, and beliefs about mathematics instruction and learning.

Recommendations

The teachers involved in the study need to be encouraged to concentrate on student thinking and learning rather than on the state mandated assessment. Long-term staff development is needed to teach content as well as teaching strategies to these teachers. It would be beneficial to these teachers to visit classrooms where the spirit of the *Standards* is demonstrated by the students and teachers.

The following related research needs to be conducted:

more research that studies the advantages and disadvantages of ability grouping in mathematics classes

more research that compares the performance on standardized tests of students from traditional and nontraditional mathematics classes

more research that studies the impact of teachers' beliefs on students' achievement.

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